Introduction

This Solutions Handbook has been designed to supplement the HP-12C Owner’s Handbook by providing a variety of applications in the financial area. Programs and/or step-by-step keystroke procedures with corresponding examples in each specific topic are explained. We hope that this book will serve as a reference guide to many of your problems and will show you how to redesign our examples to fit your specific needs.
Real Estate

Refinancing

It can be mutually advantageous to both borrower and lender to refinance an existing mortgage which has an interest rate substantially below the current market rate, with a loan at a below-market rate. The borrower has the immediate use of tax-free cash, while the lender has substantially increased debt service on a relatively small cash outlay.

To find the benefits to both borrower and lender:
1. Calculate the monthly payment on the existing mortgage.
2. Calculate the monthly payment on the new mortgage.
3. Calculate the net monthly payment received by the lender (and paid by the borrower) by adding the figure found in Step 1 to the figure found in Step 2.
4. Calculate the Net Present Value (NPV) to the lender of the net cash advanced.
5. Calculate the yield to the lender as an IRR.
6. Calculate the NPV to the borrower of the net cash received.

**Example 1:** An investment property has an existing mortgage which originated 8 years ago with an original term of 25 years, fully amortized in level monthly payments at 6.5% interest. The current balance is $133,190. Although the going current market interest rate is 11.5%, the lender has agreed to refinance the property with a $200,000, 17 year, level-monthly-payment loan at 9.5% interest.

What are the NPV and effective yield to the lender on the net amount of cash actually advanced?

What is the NPV to the borrower on this amount if he can earn a 15.25% equity yield rate on the net proceeds of the loan?

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<thead>
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<tbody>
<tr>
<td>g END</td>
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<tr>
<td>f CLEAR FIN</td>
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</tr>
<tr>
<td>17 g 12x</td>
<td>-1,080.33 Monthly payment on existing mortgage received by lender.</td>
</tr>
<tr>
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</tr>
<tr>
<td>133190 PV</td>
<td></td>
</tr>
<tr>
<td>PMT STO 0</td>
<td></td>
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</tbody>
</table>
Wrap-Around Mortgage

A mortgage loan on an income property has a remaining balance of $200,132.06. When the loan originated 8 years ago, it had a 20-year term with full amortization in level monthly payments at 6.75% interest. A lender has agreed to "wrap" a $300,000 second mortgage at 10%, with full amortization in level monthly payments over 12 years. What is the effective yield (IRR) to the lender on the net cash advanced?

<table>
<thead>
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<tbody>
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</table>

**Example 1:** A mortgage loan on an income property has a remaining balance of $200,132.06. When the loan originated 8 years ago, it had a 20-year term with full amortization in level monthly payments at 6.75% interest. A lender has agreed to "wrap" a $300,000 second mortgage at 10%, with full amortization in level monthly payments over 12 years. What is the effective yield (IRR) to the lender on the net cash advanced?
Sometimes the wrap around mortgage will have a longer payback period than the original mortgage, or a balloon payment may exist. where:

- $n_1$ = number of years remaining in original mortgage
- $PMT_1$ = yearly payment of original mortgage
- $PV_1$ = remaining balance of original mortgage
- $n_2$ = number of years in wrap-around mortgage
- $PMT_2$ = yearly payment of wrap-around mortgage
- $PV_2$ = total amount of wrap-around mortgage
- $BAL$ = balloon payment

### Table

<table>
<thead>
<tr>
<th>Operation</th>
<th>Calculation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>CLEAR FIN</td>
<td></td>
<td>Monthly interest rate (into $i$).</td>
</tr>
<tr>
<td>20 ENTER</td>
<td></td>
<td>Loan amount (into $PV$).</td>
</tr>
<tr>
<td>8 - 9 12x</td>
<td></td>
<td>Monthly payment on existing mortgage (calculated).</td>
</tr>
<tr>
<td>6.75 g 12+</td>
<td>0.56</td>
<td>Monthly interest on wrap-around.</td>
</tr>
<tr>
<td>PMT STO 0</td>
<td>-2,031.55</td>
<td>Amount of wrap-around (into $PV$).</td>
</tr>
<tr>
<td>10 g 12+</td>
<td>0.83</td>
<td>Monthly payment on wrap-around (calculated).</td>
</tr>
<tr>
<td>300000 CHS PV</td>
<td>-300,000.00</td>
<td>Net monthly payment received (into $PMT$).</td>
</tr>
<tr>
<td>PMT</td>
<td>3.585.23</td>
<td>Net cash advanced (into $PV$).</td>
</tr>
<tr>
<td>RCL 0 + PMT</td>
<td>1,553.69</td>
<td>Nominal yield ($IRR$) to lender (calculated).</td>
</tr>
<tr>
<td>RCL PV</td>
<td>-99,867.94</td>
<td></td>
</tr>
<tr>
<td>200132.06 + PV</td>
<td>15.85</td>
<td></td>
</tr>
</tbody>
</table>

![Diagram of wrap around mortgage](image)
**Example 2**: A customer has an existing mortgage with a balance of $125,010, a remaining term of 200 months, and a $1051.61 monthly payment. He wishes to obtain a $200,000, 9 1/2% wrap-around with 240 monthly payments of $1681.71 and a balloon payment at the end of the 240th month of $129,963.35. If you, as a lender, accept the proposal, what is your rate of return?

![Diagram showing cash flows and calculations]

<table>
<thead>
<tr>
<th>Keystrokes</th>
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</thead>
<tbody>
<tr>
<td>g END</td>
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</tr>
<tr>
<td>f CLEAR FIN</td>
<td>-74,990.00 Net investment.</td>
</tr>
<tr>
<td>200000 CHS ENTER</td>
<td>125010 + g CF_0</td>
</tr>
<tr>
<td>1051.61 CHS ENTER</td>
<td>630.10 Net cash flow received by lender.</td>
</tr>
<tr>
<td>g CF_j 99 g N_j</td>
<td>The above cash flow occurs 200 times.</td>
</tr>
<tr>
<td>x ÷ y g CF_j</td>
<td>1,681.71 Next cash flow received by lender.</td>
</tr>
<tr>
<td>x ÷ y g N_j</td>
<td>39.00 Cash flow occurs 39 times.</td>
</tr>
<tr>
<td>x ÷ y 129963.35 +</td>
<td>131,645.06 Final cash flow.</td>
</tr>
<tr>
<td>g CF_j</td>
<td></td>
</tr>
<tr>
<td>f IRR 12 x</td>
<td>11.84 Rate of return to lender.</td>
</tr>
</tbody>
</table>

$125010 \rightarrow 240 \text{ mos.} \rightarrow 129963.35$
If you, as a lender, know the yield on the entire transaction, and you wish to obtain the payment amount on the wrap-around mortgage to achieve this yield, use the following procedure. Once the monthly payment is known, the borrower’s periodic interest rate may also be determined.

1. Press the $g$ END and press $f$ CLEAR FIN.
2. Key in the remaining periods of the original mortgage and press $n$.
3. Key in the desired annual yield and press $g$ 12+$+$.
4. Key in the monthly payment to be made by the lender on the original mortgage and press CHS PMT.
5. Press PV.
6. Key in the net amount of cash advanced and press $+$ CHS PV.
7. Key in the total term of the wrap-around mortgage and press $n$.
8. If a balloon payment exists, key it in and press FV.
9. Press PMT to obtain the payment amount necessary to achieve the desired yield.
10. Key in the amount of the wrap-around mortgage and press CHS PV i to obtain the borrower’s periodic interest rate.

**Example 3:** Your firm has determined that the yield on a wrap-around mortgage should be 12% annually. In the previous example, what monthly payment must be received to achieve this yield on a $200,000 wrap-around? What interest rate is the borrower paying?

<table>
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<tbody>
<tr>
<td>200 12 g 12+$</td>
<td>1051.61 CHS PMT</td>
</tr>
<tr>
<td>PV 74990 $+$ CHS PV</td>
<td>-165,776.92</td>
</tr>
<tr>
<td>240 n 129963.35 FV PMT</td>
<td>1,693.97</td>
</tr>
<tr>
<td>2000 CHS PV i</td>
<td>9.58</td>
</tr>
</tbody>
</table>

Number of periods and monthly interest rate.

Present value of payments plus cash advanced.

Monthly payment received by lender.

Annual interest rate paid by borrower.
Income Property Cash Flow Analysis

Before-Tax Cash Flows
The before-tax cash flows applicable to real estate analysis and problems are:

- Potential Gross Income
- Effective Gross Income
- Net Operating Income (also called Net Income Before Recapture.)
- Cash Throw-off to Equity (also called Gross Spendable Cash)

The derivation of these cash flows follows a set sequence:

1. Calculate Potential Gross Income by multiplying the rent per unit times the number of units, times the number of rental payments periods per year. This gives the rental income the property would generate if it were fully occupied.
2. Deduct Allowance for Vacancy and Rental Loss. This is usually expressed as a percentage. The result is Rent Collections (which is also Effective Gross Income if there is no "Other Income").
3. Add "Other Income" such as receipts from concessions (laundry equipment, etc.), produced from sources other than the rental office space. This is Effective Gross Income.
4. Deduct Operating Expenses. These are expenditures the landlord-investor must make, by contract or custom, to preserve the property and keep in capable of producing the gross income. The result is the Net Operating Income.
5. Deduct Annual Debt Service on the mortgage. This produces Cash Throw-Off to Equity.

Thus:

Effective Gross Income = Potential Gross Income - Vacancy Loss + Other Income.

Net Operating Income = Effective Gross Income - Operating Expenses.

Cash Throw-Off = Net Operating Income - Annual Dept Service.

Example: A 60-unit apartment building has rentals of $250 per unit per month. With a 5% vacancy rate, the annual operating cost is $76,855. The property has just been financed with a $700,000 mortgage, fully amortized in a level monthly payments at 11.5% over 20 years.

a. What is the Effective Gross Income?
b. What is the Net Operating Income?
c. What is the Cash Throw-Off to Equity?
Before-Tax Reversions (Resale Proceeds)

The reversion receivable at the end of the income projection period is usually based on forecast or anticipated resale of the property at that time. The before tax reversion amount applicable to real estate analysis and problems are:

- Sale Price.
- Cash Proceeds of Resale.
- Outstanding Mortgage Balance.
- Net Cash Proceeds of Resale to Equity.

The derivation of these reversions are as follows:

1. Forecast or estimate Sales Price. Deduct sales and Transaction Costs. The result is the Proceeds of Resale.
2. Calculate the Outstanding Balance of the Mortgage at the end of the Income Projection Period and subtract it from Proceeds of Resale. The result is net Cash Proceeds of Resale.

Thus:

\[
\text{Cash Proceeds of Resale} = \text{Sales Price} - \text{Transaction Costs.}
\]

\[
\text{Net Cash Proceeds of Resale} = \text{Cash Proceeds of Resale} - \text{Outstanding Mortgage Balance.}
\]

**Example:** The apartment property in the preceding example is expected to be resold in 10 years. The anticipated resale price is $800,000. The

<table>
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<tr>
<td>9</td>
<td>END</td>
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<tr>
<td>f CLEAR</td>
<td>FIN</td>
</tr>
<tr>
<td>60</td>
<td>ENTER</td>
</tr>
<tr>
<td>250 x 12 x</td>
<td></td>
</tr>
<tr>
<td>5 %</td>
<td>9,000.00</td>
</tr>
<tr>
<td>-</td>
<td>171,000.00</td>
</tr>
<tr>
<td>76855 -</td>
<td>94,145.00</td>
</tr>
<tr>
<td>20 g 12x</td>
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<tr>
<td>11.5 g 12+</td>
<td>-89,580.09</td>
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<tr>
<td>700000 PV</td>
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</tr>
<tr>
<td>PMT 12 x</td>
<td>4,564.91</td>
</tr>
<tr>
<td>+</td>
<td></td>
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</table>
transaction costs are expected to be 7% of the resale price. The mortgage is the same as that indicated in the preceding example.

- What will the Mortgage Balance be in 10 years?
- What are the Cash Proceeds of Resale and Net Cash Proceeds of Resale?

<table>
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<tr>
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<td>240.00 Mortgage term.</td>
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<td>11.5 9 12x</td>
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<tr>
<td>PMT</td>
<td>-7,465.01 Monthly payment.</td>
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<tr>
<td>10 9 12x</td>
<td>120.00 Projection period.</td>
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<td>FV</td>
<td>-530,956.57 Mortgage balance in 10 years.</td>
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<td>800000 ENTER</td>
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<tr>
<td>7 %</td>
<td>56,000.00 Transaction costs.</td>
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<tr>
<td>-</td>
<td>744,000.00 Cash Proceeds of Resale.</td>
</tr>
<tr>
<td>+</td>
<td>213,043.43 Net Cash Proceeds of Resale.</td>
</tr>
</tbody>
</table>

**After-Tax Cash Flows**

The After-Tax Cash Flow (ATCF) is found for each year by deducting the Income Tax Liability for that year from the Cash Throw Off.

Where:

- **Taxable Income** = 
  Net Operating Income - interest - depreciation.

- **Tax Liability** = 
  Taxable Income x Marginal Tax Rate.

- **After Tax Cash Flow** = 
  Cash Throw Off - Tax Liability.

The After-Tax Cash Flow for the initial and successive years may be calculated by the following HP-12C program. This program calculates the Net Operating Income using the Potential Gross Income, operational cost and vacancy rate. The Net Operating Income is readjusted each year from the growth rates in Potential Gross Income and operational costs.

The user is able to change the method of finding the depreciation from declining balance to straight line. To make the change, key in [f] SL at line 32 of the program in place of [f] DB.
<table>
<thead>
<tr>
<th>KEYSTROKES</th>
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</thead>
<tbody>
<tr>
<td>f [P/R]</td>
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<td>f CLEAR PRGM</td>
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<td>01- 0</td>
</tr>
<tr>
<td>n</td>
<td>02- 11</td>
</tr>
<tr>
<td>STO 1</td>
<td>03- 44 1</td>
</tr>
<tr>
<td>RCL 7</td>
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</tr>
<tr>
<td>EEX</td>
<td>05- 26</td>
</tr>
<tr>
<td>2</td>
<td>06- 2</td>
</tr>
<tr>
<td>+</td>
<td>07- 10</td>
</tr>
<tr>
<td>STO 7</td>
<td>08- 44 7</td>
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<tr>
<td>1</td>
<td>09- 1</td>
</tr>
<tr>
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</tr>
<tr>
<td>1</td>
<td>11- 1</td>
</tr>
<tr>
<td>2</td>
<td>12- 2</td>
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<tr>
<td>f [AMORT]</td>
<td>13- 42 11</td>
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<tr>
<td>n</td>
<td>16- 11</td>
</tr>
<tr>
<td>RCL [i]</td>
<td>17- 45 12</td>
</tr>
<tr>
<td>RCL 6</td>
<td>18- 45 6</td>
</tr>
<tr>
<td>[i]</td>
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</tr>
<tr>
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<td>20- 33</td>
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<tr>
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<td>21- 44 6</td>
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<tr>
<td>R↓</td>
<td>22- 33</td>
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<tr>
<td>RCL [PV]</td>
<td>23- 45 13</td>
</tr>
<tr>
<td>RCL 4</td>
<td>24- 45 4</td>
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<tr>
<td>[PV]</td>
<td>25- 13</td>
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<tr>
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<td>26- 33</td>
</tr>
<tr>
<td>Command</td>
<td>Description</td>
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<td>---------</td>
<td>-------------</td>
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<tr>
<td><strong>STO</strong> 4</td>
<td>27 - 44 4</td>
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<td><strong>R↓</strong></td>
<td>28 - 33</td>
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<tr>
<td><strong>g</strong>&lt;sub&gt;x=0&lt;/sub&gt;</td>
<td>29 - 43 35</td>
</tr>
<tr>
<td><strong>g</strong>&lt;sub&gt;GTO&lt;/sub&gt; 36</td>
<td>30 - 43, 33 36</td>
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<tr>
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<td>31 - 45 1</td>
</tr>
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</tr>
<tr>
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<tr>
<td>0</td>
<td>34 - 0</td>
</tr>
<tr>
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<td>35 - 43, 33 17</td>
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<td>36 - 11</td>
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<tr>
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<td>38 - 45 8</td>
</tr>
<tr>
<td>%</td>
<td>39 - 25</td>
</tr>
<tr>
<td><strong>STO</strong>&lt;sub&gt;+&lt;/sub&gt; 2</td>
<td>40 - 44 40 2</td>
</tr>
<tr>
<td><strong>R↓</strong></td>
<td>41 - 33</td>
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<tr>
<td>%</td>
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</tr>
<tr>
<td>-</td>
<td>44 - 30</td>
</tr>
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<td>45 - 45 3</td>
</tr>
<tr>
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</tr>
<tr>
<td>%</td>
<td>47 - 25</td>
</tr>
<tr>
<td><strong>STO</strong>&lt;sub&gt;+&lt;/sub&gt; 3</td>
<td>48 - 44 40 3</td>
</tr>
<tr>
<td><strong>R↓</strong></td>
<td>49 - 33</td>
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<td>50 - 30</td>
</tr>
<tr>
<td>1</td>
<td>51 - 1</td>
</tr>
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</tr>
<tr>
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<td>53 - 44 20 0</td>
</tr>
<tr>
<td>-</td>
<td>54 - 30</td>
</tr>
</tbody>
</table>
1. Press $g$ END and press $f$ CLEAR REG.

2. Key in loan values:
   - Key in annual interest rate and press $g$ 12EX
   - Key in principal to be paid and press PV
   - Key in monthly payment and press CHS PMT
     (If any of the values are not known, they should be solved for.)

3. Key in Potential Gross Income (PGI) and press STO 2.

5. Key in depreciable value and press \( \text{STO} \) 4.
6. Key in depreciable life and press \( \text{STO} \) 5.
7. Key in factor (for declining balance only) and press \( \text{STO} \) 6.
8. Key in the Marginal Tax Rate (as a percentage) and press \( \text{STO} \) 7.
9. Key in the growth rate in Potential Gross Income (0 for no growth) and press \( \text{STO} \) 8.
10. Key in the growth rate in operational cost (0 if no growth) and press \( \text{STO} \) 9.
11. Key in the vacancy rate (0 for no vacancy rate) and press \( \text{STO} \) • 0.
12. Key in the desired depreciation function at line 32 in the program.
13. Press [R/S] to compute \( ATCF \). The display will pause showing the year and then will stop with the \( ATCF \) for that year. The Y-register contains the year.

**Example 1:** A triplex was recently purchased for $100,000 with a 30-year loan at 12.25% and a 20% down payment. Not including a 5% annual vacancy rate, the potential gross income is $9,900 with an annual growth rate of 6%. Operating expenses are $3,291.75 with a 2.5% growth rate. The depreciable value is $75,000 with a projected useful life of 20 years. Assuming a 125% declining balance depreciation, what are the After-Tax Cash Flows for the first 10 years if the investors Marginal Tax Rate is 35%?

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<td>80,000.00</td>
</tr>
<tr>
<td>( \text{FIN} ) ( \text{FIN} )</td>
<td>Mortgage amount.</td>
</tr>
<tr>
<td>100000 ( \text{ENTER} ) 20 % ( \text{PV} )</td>
<td>1.02 Monthly interest rate.</td>
</tr>
<tr>
<td>12.25 ( g ) 12x</td>
<td>360 Mortgage term.</td>
</tr>
<tr>
<td>30 ( g ) 12x</td>
<td>-838.32 Monthly payment.</td>
</tr>
<tr>
<td>( \text{PMT} ) 9900 ( \text{STO} ) 2</td>
<td>9,900.00 Potential Gross Income.</td>
</tr>
<tr>
<td>3291.75 ( \text{STO} ) 3</td>
<td>3,291.75 1st year operating cost.</td>
</tr>
<tr>
<td>75000 ( \text{STO} ) 4</td>
<td>75,000.00 Depreciable value.</td>
</tr>
<tr>
<td>20 ( \text{STO} ) 5</td>
<td>20.00 Useful life.</td>
</tr>
</tbody>
</table>
**Example 2:** An office building was purchased for $1,400,000. The value of depreciable improvements is $1,200,000.00 with a 35 year economic life. Straight line depreciation will be used. The property is financed with a $1,050,000 loan. The terms of the loan are 9.5% interest and $9,173.81 monthly payments for 25 years. The office building generates a Potential Gross Income of $175,200.00 which grows at a 3.5% annual rate. The operating cost is $40,296.00 with a 1.6% annual growth rate. Assuming a Marginal Tax Rate of 50% and a vacancy rate of 7%, what are the After-Tax Cash Flows for the first 5 years?

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</tr>
</thead>
<tbody>
<tr>
<td>125 [STO] 6</td>
<td>125.00 Decline in balance factor.</td>
</tr>
<tr>
<td>35 [STO] 7</td>
<td>35.00 Marginal Tax Rate.</td>
</tr>
<tr>
<td>6 [STO] 8</td>
<td>6.00 Potential Gross Income growth rate.</td>
</tr>
<tr>
<td>2.5 [STO] 9</td>
<td>2.50 Operating cost growth.</td>
</tr>
<tr>
<td>5 [STO] 0</td>
<td>5.00 Vacancy rate.</td>
</tr>
<tr>
<td>R/S</td>
<td>1.00 Year 1 ( ATCF_1 )</td>
</tr>
<tr>
<td>R/S</td>
<td>-1,020.88</td>
</tr>
<tr>
<td>R/S</td>
<td>2.00 Year 2 ( ATCF_2 )</td>
</tr>
<tr>
<td>R/S</td>
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</tr>
<tr>
<td>R/S</td>
<td>3.00 Year 3 ( ATCF_3 )</td>
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<tr>
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<td>8.00 Year 8 ( ATCF_8 )</td>
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<td>R/S</td>
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<td>10.00 Year 10 ( ATCF_{10} )</td>
</tr>
<tr>
<td>1050000 PV</td>
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</tr>
<tr>
<td>9173.81 CHS PMT</td>
<td>175,200.00 Potential Gross Income.</td>
</tr>
<tr>
<td>9.5 [g] 12÷</td>
<td></td>
</tr>
</tbody>
</table>

Keystrokes: 
- CLEAR
- [f] CLEAR REG
- 1050000 [PV]
- 9173.81 CHS PMT
- 9.5 [g] 12÷
The After-Tax Net Cash Proceeds of Resale (ATNCPR) is the after-tax reversion to equity; generally, the estimated resale price of the property less commissions, outstanding debt and any tax claim.

The After-Tax Net Cash Proceeds can be found using the HP-12C program which follows. In calculating the owner's income tax liability on resale, this program assumes that the owner elects to have his capital gain taxed at 40% of his Marginal Tax Rate. This assumption is in accordance with a 1978 Federal tax ruling. (*Federal Taxes, code sec. 1202 (32,036))

This program uses declining balance depreciation to find the amount of depreciation from purchase to sale. This amount is used to determine the excess depreciation (which is equal to the amount of actual depreciation minus the amount of the straight line depreciation).

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<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Result</th>
</tr>
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<td>12x</td>
<td>1st year operating cost.</td>
<td>40,296.00</td>
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<tr>
<td>STO</td>
<td>Depreciable value.</td>
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<tr>
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<td>Depreciable life.</td>
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<td>50</td>
<td>Marginal tax rate.</td>
<td>50.00</td>
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<td>STO</td>
<td>Potential Gross Income</td>
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<tr>
<td>1.6</td>
<td>Operating cost growth rate.</td>
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<td>Vacancy rate.</td>
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<td>Go to dep. step.</td>
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<tr>
<td>P/R</td>
<td>Change to SL.</td>
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<tr>
<td>SL</td>
<td>Year 1 ATCF</td>
<td>18,021.07</td>
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<td>P/R</td>
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<td>R/S</td>
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<td>22,048.90</td>
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<tr>
<td>R/S</td>
<td>Year 4 ATCF</td>
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<tr>
<td>R/S</td>
<td>Year 5 ATCF</td>
<td>26,234.69</td>
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</table>
The user may change to a different depreciation method by keying in the desired function at line 35 in place of \[ T \text{ DB} \].

<table>
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<th>DISPLAY</th>
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<td>[ STO \text{ 2} ]</td>
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<td>[ \div ]</td>
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<td>[ - ]</td>
<td>05- 30</td>
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<tr>
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<tr>
<td>[ - ]</td>
<td>07- 30</td>
</tr>
<tr>
<td>[ \times ]</td>
<td>08- 48</td>
</tr>
<tr>
<td>[ 4 ]</td>
<td>09- 4</td>
</tr>
<tr>
<td>[ \times ]</td>
<td>10- 20</td>
</tr>
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<tr>
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<tr>
<td>[ \text{RND} ]</td>
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<tr>
<td>[ PMT ]</td>
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<tr>
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<tr>
<td>[ g \text{ 12x} ]</td>
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<td>[ FV ]</td>
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<td>[ RCL \text{ 3} ]</td>
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<td>[ PV ]</td>
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<tr>
<td>[ RCL \text{ 4} ]</td>
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<tr>
<td>[ n ]</td>
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<tr>
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<tr>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>1</td>
<td>25-</td>
</tr>
<tr>
<td>RCL 2</td>
<td>26-</td>
</tr>
<tr>
<td>f SL</td>
<td>27-</td>
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<td>RCL 2</td>
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</tr>
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<td>x</td>
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</tr>
<tr>
<td>x^2</td>
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<td>31-</td>
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<td>x</td>
<td>32-</td>
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<tr>
<td>STO + 1</td>
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<td>34-</td>
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<tr>
<td>f DB</td>
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<td>x≥y</td>
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<tr>
<td>-</td>
<td>38-</td>
</tr>
<tr>
<td>STO + 1</td>
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<td>RCL 6</td>
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<td>EEEX</td>
<td>41-</td>
</tr>
<tr>
<td>2</td>
<td>42-</td>
</tr>
<tr>
<td>+</td>
<td>43-</td>
</tr>
<tr>
<td>RCL 1</td>
<td>44-</td>
</tr>
<tr>
<td>x</td>
<td>45-</td>
</tr>
<tr>
<td>RCL 0</td>
<td>46-</td>
</tr>
<tr>
<td>+</td>
<td>47-</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>48-43</td>
</tr>
<tr>
<td>f P/R</td>
<td></td>
</tr>
</tbody>
</table>

REGISTERS
n: Used
i: Used
PV: Used
PMT: Used
FV: Used
R0: Used
R1: Used
R2: Desired yr.
1. Key in the program and press \[ T \text{ CLEAR REG} \].
2. Key in the loan values:
   - Key in annual interest rate and press \[ g [12+] \]
   - Key in mortgage amount and press \[ PV \]
   - Key in monthly payment and press \[ CHS PMT \]
     (If any of the values are unknown, they should be solved for.)
3. Key in depreciable value and press \[ STO 3 \].
4. Key in depreciable life in years and press \[ STO 4 \].
5. Key in accelerated depreciation factor for the declining balance method and press \[ STO 5 \].
6. Key in your Marginal Tax Rate as a percentage and press \[ STO 6 \].
7. Key in the purchase price and press \[ ENTER \].
8. Key in the sale price and press \[ ENTER \].
9. Key in the % commission charged on the sale and press \[ ENTER \].
   "If a dollar value is desired instead of a commission rate, key in \[ g \text{ END} \],
   which does not affect the register values, at line 04 of the program.
10. Key in the number of years after purchase and press \[ R/S \].

**Example 1:** An apartment complex, purchased for $900,000 ten years ago, is sold for $1,750,000. The closing cost are 8% of the sale price and the income tax rate is 48%.

A $700,000 loan for 20 years at 9.5% annual interest was used to purchase the complex. When it was purchased the depreciable value was $750,000 with a useful life of 25 years. Using 125% declining balance depreciation, what are the After-Tax Net Cash Proceeds in year 10?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ g \text{ END} ]</td>
<td>0.00</td>
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<tr>
<td>[ T \text{ CLEAR REG} ]</td>
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</tr>
<tr>
<td>PV</td>
<td>700,000.00</td>
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<tr>
<td>----</td>
<td>------------</td>
</tr>
<tr>
<td>g</td>
<td>9.5</td>
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<tr>
<td>g</td>
<td>12+</td>
</tr>
<tr>
<td>g</td>
<td>12x</td>
</tr>
<tr>
<td>PMT</td>
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<tr>
<td>STO</td>
<td>750,000.00</td>
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<tr>
<td>STO</td>
<td>25</td>
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<td>STO</td>
<td>125</td>
</tr>
<tr>
<td>STO</td>
<td>48</td>
</tr>
<tr>
<td>STO</td>
<td>6</td>
</tr>
<tr>
<td>ENTER</td>
<td>900,000.00</td>
</tr>
<tr>
<td>ENTER</td>
<td>1,750,000.00</td>
</tr>
<tr>
<td>ENTER</td>
<td>8.00</td>
</tr>
<tr>
<td>R/S</td>
<td>911,372.04</td>
</tr>
</tbody>
</table>
Lending

Loan With a Constant Amount Paid Towards Principal

This type of loan is structured such that the principal is repaid in equal installments with the interest paid in addition. Therefore each periodic payment has a constant amount applied toward the principle and a varying amount of interest.

Loan Reduction Schedule

If the constant periodic payment to principal, annual interest rate, and loan amount are known, the total payment, interest portion of each payment, and remaining balance after each successive payment may be calculated as follows:

1. Key in the constant periodic payment to principal and press \[^{\text{STO}}\].
2. Key in periodic interest rate and press \[^{\text{ENTER}}\] \[^{\text{ENTER}}\] \[^{\text{ENTER}}\].
3. Key in the loan amount. If you wish to skip to another time period, press \[^{\text{RCL}}\] \[^{x \div y}\] \[^{\text{x}}\] \[^{\text{RCL}}\]. Then key in the number of payments to be skipped, and press \[^{\text{RCL}}\] \[^{x \div y}\] \[^{\text{x}}\] \[^{\text{RCL}}\].
4. Press \[^{\text{x \div y}}\] \[^{\%}\] to obtain the interest portion of the payment.
5. Press \[^{\text{RCL}}\] \[^{0}\] \[^{+}\] to obtain the total payment.
6. Press \[^{\text{CLX}}\] \[^{\text{RCL}}\] \[^{0}\] \[^{–}\] to obtain the remaining balance of the loan.
7. Return to step 4 for each successive payment.

Example 1: A $60,000 land loan at 10% interest calls for equal semi-annual principal payments over a 6-year maturity. What is the loan reduction schedule for the first year? (Constant payment to principal is $5000 semi-annually). What is the fourth year's schedule (skip 4 payments)?

<table>
<thead>
<tr>
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<th>Display</th>
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</thead>
<tbody>
<tr>
<td>5000 [^{\text{STO}}] 0</td>
<td>5.00</td>
</tr>
<tr>
<td>10 [^{\text{ENTER}}] 2 [^{\text{÷}}] [^{\text{ENTER}}] [^{\text{ENTER}}]</td>
<td>3,000.00</td>
</tr>
<tr>
<td>60000 [^{\times \div y}] [^{%}]</td>
<td>8,000.00</td>
</tr>
</tbody>
</table>
Add-On Interest Rate Converted to APR

An add-on interest rate determines what portion of the principal will be added on for repayment of a loan. This sum is then divided by the number of months in a loan to determine the monthly payment. For example, a 10% add-on rate for 36 months on $3000 means add one-tenth of $3000 for 3 years (300 x 3) - usually called the “finance charge” - for a total of $3900. The monthly payment is $3900/36.

This keystroke procedure converts an add-on interest rate to a annual percentage rate when the add-on rate and number of months are known.

1. Press \( \text{g} \) \( \text{END} \) and press \( \text{f} \) \( \text{CLEAR} \) \( \text{FIN} \).
2. Key in the number of months in loan and press \( \text{n} \) \( \text{ENTER} \) \( \text{RCL} \) \( \text{g} \) \( 12 \times \).
3. Key in the add-on rate and press \( \text{x} \).
4. Key in the amount of the loan and press \( \text{PV} \) (“Positive for cash received; negative for cash paid out.”) \( \text{x} \) \( \text{y} \) \( \% \) \( + \).
5. Press \( \text{x} \) \( \text{y} \) \( \% \) \( + \) \( \text{CHS} \) \( \text{PMT} \).
6. Press \( \frac{1}{2} \) \( 12 \) \( \times \) to obtain the APR.
**Example 1:** Calculate the APR and monthly payment of a 12% $1000 add-on loan which has a life of 18 months.

<table>
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<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>END</td>
<td></td>
</tr>
<tr>
<td>CLEAR FIN</td>
<td></td>
</tr>
<tr>
<td>18 n ENTER</td>
<td>1,180.00 Amount of loan.</td>
</tr>
<tr>
<td>RCL g 12x 12 x</td>
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</tr>
<tr>
<td>1000 PV x y % +</td>
<td></td>
</tr>
<tr>
<td>x y + CHS PMT</td>
<td>-65.56 Monthly payment.</td>
</tr>
<tr>
<td>i 12 x</td>
<td>21.64 Annual Percentage Rate.</td>
</tr>
</tbody>
</table>

**APR Converted to Add-On Interest Rate.**

Given the number of months and annual percentage rate, this procedure calculates the corresponding add-on interest rate.

1. Press END and press CLEAR FIN.
2. Enter the following information:
   a. Key in number of months of loan and press n.
   b. Key in APR and press 12+.
   c. Key in 100 and press PV PMT.
3. Press RCL PV RCL n + + CHS 12 x to obtain the add-on rate.

**Example 1:** What is the equivalent add-on rate for an 18 month loan with an APR of 14%.

<table>
<thead>
<tr>
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<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>END</td>
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</tr>
<tr>
<td>CLEAR FIN</td>
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<tr>
<td>18 n 14 g 12+</td>
<td>7.63 Add-On Interest Rate.</td>
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<tr>
<td>100 PV PMT RCL PV</td>
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<tr>
<td>RCL n + + CHS</td>
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</tr>
<tr>
<td>12 x</td>
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</tbody>
</table>
Add-On Rate Loan with Credit Life.

This HP-12C program calculates the monthly payment amount, credit life amount (an optional insurance which cancels any remaining indebtedness at the death of the borrower), total finance charge, and annual percentage rate (APR) for an add-on interest rate (AIR) loan. The monthly payment is rounded (in normal manner) to the nearest cent. If other rounding techniques are used, slightly different results may occur.

<table>
<thead>
<tr>
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<th>Display</th>
</tr>
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<tbody>
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<td>f P/R</td>
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<td>g END</td>
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<td>02- 1</td>
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<td>+</td>
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<tr>
<td>x</td>
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<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Key</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td><code>g</code></td>
<td>FRAC</td>
</tr>
<tr>
<td><code>g</code></td>
<td>x=0</td>
</tr>
<tr>
<td><code>g</code></td>
<td>GTO 61</td>
</tr>
<tr>
<td>RCL 5</td>
<td></td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>RND</td>
</tr>
<tr>
<td>STO 5</td>
<td></td>
</tr>
<tr>
<td>R/S 5</td>
<td></td>
</tr>
<tr>
<td>R/S</td>
<td></td>
</tr>
<tr>
<td>RCL 5</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>PV</td>
</tr>
<tr>
<td>x,y</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>RCL 3</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
</tr>
<tr>
<td>CHS</td>
<td></td>
</tr>
<tr>
<td>R/S</td>
<td></td>
</tr>
<tr>
<td>RCL 5</td>
<td></td>
</tr>
<tr>
<td>RCL 3</td>
<td></td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td></td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>RCL 0</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td></td>
</tr>
<tr>
<td>RCL g 12+</td>
<td></td>
</tr>
<tr>
<td><code>g</code></td>
<td>GTO 00</td>
</tr>
<tr>
<td><code>t</code> P/R</td>
<td></td>
</tr>
</tbody>
</table>
1. Key in the program.

2. Press \[
\text{F} \quad \text{CLEAR} \quad \text{FIN}.
\]

3. Key in the number of monthly payments in the loan and press \[
\text{STO} \quad 0.
\]

4. Key in the annual add-on interest rate as a percentage and press \[
\text{STO} \quad 1.
\]

5. Key in the credit life as a percentage and press \[
\text{STO} \quad 2.
\]

6. Key in the loan amount and press \[
\text{STO} \quad 3.
\]

7. Press \[
\text{R/S}
\] to find the monthly payment amount.

8. Press \[
\text{R/S}
\] to obtain the amount of credit life.

9. Press \[
\text{R/S}
\] to calculate the total finance charge.

10. Press \[
\text{R/S}
\] to calculate the annual percentage rate.

11. For a new loan return to step 3.

**Example 1:** You wish to quote a loan on a $3100 balance, payable over 36 months at an add-on rate of 6.75%. Credit life (CL) is 1%. What are the monthly payment amount, credit life amount, total finance charge, and APR?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
</table>
| \[
\text{F} \quad \text{CLEAR} \quad \text{FIN}
\] | \[
36.00 \quad \text{Months.}
\] |
| \[
36 \quad \text{STO} \quad 0
\] | \[
6.75 \quad \text{Add-on interest rate.}
\] |
| \[
6.75 \quad \text{STO} \quad 1
\] | \[
1.00 \quad \text{Credit life (%).}
\] |
| \[
3100 \quad \text{STO} \quad 3
\] | \[
3100.00 \quad \text{Loan.}
\] |
| \[
\text{R/S}
\] | \[
-107.42 \quad \text{Monthly payment.}
\] |
| \[
\text{R/S}
\] | \[
116.02 \quad \text{Credit life.}
\] |
Interest Rebate - Rule of 78's

This procedure finds the unearned interest rebate, as well as the remaining principal balance due for a prepaid consumer loan using the Rule of 78's. The known values are the current installment number, the total number of installments for which the loan was written, and the total finance charge (amount of interest). The information is entered as follows:

1. Key in number of months in the loan and press \[ \text{STO} \ 1 \].
2. Key in payment number when prepayment occurs and press \[ \text{STO} \ 2 \] 1 +.
3. Key in total finance charge and press \[ \text{RCL} \ 1 \ \text{ENTER} \ x \ \text{RCL} \ 2 \ x \] to obtain the unearned interest (rebate).
4. Key in periodic payment amount and press \[ \text{RCL} \ 2 \ x \ x \div \ y \] - to obtain the amount of principal outstanding.

**Example 1:** A 30 month $1000 loan having a finance charge of $180, is being repaid at $39.33 per month. What is the rebate and balance due after the 25th regular payment?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 \text{STO} 1</td>
<td></td>
</tr>
<tr>
<td>25 \text{STO} 2</td>
<td></td>
</tr>
<tr>
<td>1 + 180</td>
<td></td>
</tr>
<tr>
<td>\text{RCL} 1 \text{ENTER}</td>
<td></td>
</tr>
<tr>
<td>\text{x} \ \text{RCL} 1 +</td>
<td></td>
</tr>
<tr>
<td>+ \text{RCL} 2 \text{x}</td>
<td></td>
</tr>
<tr>
<td>39.33 \text{RCL} 2 \text{x}</td>
<td></td>
</tr>
<tr>
<td>\text{x} \div \text{y} -</td>
<td></td>
</tr>
</tbody>
</table>

The following HP-12C program can be used to evaluate the previous example.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>\f P/R</td>
<td></td>
</tr>
<tr>
<td>\f \text{CLEAR PRGM}</td>
<td></td>
</tr>
<tr>
<td>Operation</td>
<td>Register</td>
</tr>
<tr>
<td>-----------</td>
<td>----------</td>
</tr>
<tr>
<td>STO 0</td>
<td>01</td>
</tr>
<tr>
<td>R↓</td>
<td>02</td>
</tr>
<tr>
<td>STO 2</td>
<td>03</td>
</tr>
<tr>
<td>R↓</td>
<td>04</td>
</tr>
<tr>
<td>STO 1</td>
<td>05</td>
</tr>
<tr>
<td>RCL 2</td>
<td>06</td>
</tr>
<tr>
<td>-</td>
<td>07</td>
</tr>
<tr>
<td>STO 2</td>
<td>08</td>
</tr>
<tr>
<td>+</td>
<td>09</td>
</tr>
<tr>
<td>RCL 0</td>
<td>10</td>
</tr>
<tr>
<td>x</td>
<td>11</td>
</tr>
<tr>
<td>RCL 1</td>
<td>12</td>
</tr>
<tr>
<td>ENTER</td>
<td>13</td>
</tr>
<tr>
<td>x</td>
<td>14</td>
</tr>
<tr>
<td>RCL 1</td>
<td>15</td>
</tr>
<tr>
<td>+</td>
<td>16</td>
</tr>
<tr>
<td>RCL 2</td>
<td>17</td>
</tr>
<tr>
<td>x</td>
<td>18</td>
</tr>
<tr>
<td>R/S</td>
<td>19</td>
</tr>
<tr>
<td>RCL 2</td>
<td>20</td>
</tr>
<tr>
<td>x</td>
<td>21</td>
</tr>
<tr>
<td>x≥y</td>
<td>22</td>
</tr>
<tr>
<td>-</td>
<td>23</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>24</td>
</tr>
<tr>
<td>f P/R</td>
<td>25</td>
</tr>
</tbody>
</table>

**REGISTERS**

n: Unused

I: Unused
1. Key in the program.
2. Key in the number of months in the loan and press \( \text{ENTER} \).
3. Key in the payment number when prepayment occurs and press \( \text{ENTER} \).
4. Key in the total finance charge and press \( \text{R/S} \) to obtain the unearned interest (rebate).
5. Key in the periodic payment amount and press \( \text{R/S} \) to find the amount of principal outstanding.
6. For a new case return to step 2.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 ENTER</td>
<td>5.81 Rebate</td>
</tr>
<tr>
<td>25 ENTER</td>
<td></td>
</tr>
<tr>
<td>180 R/S</td>
<td>190.84 Outstanding principal.</td>
</tr>
</tbody>
</table>

**Graduated Payment Mortgages**

The Graduated Payment Mortgage is designed to meet the needs of young home buyers who currently cannot afford high mortgage payments, but who have the potential of increasing earning in the years on come.

Under the Graduated Payment Mortgage plan, the payments increase by a fixed percentage at the end of each year for a specified number of years. Thereafter, the payment amount remains constant for remaining life of the mortgage.

The result is that the borrower pays a reduced payment (a payment which is less than a traditional mortgage payment) in the early years, and in the later years makes larger payments than he would with a traditional loan. Over the entire term of the mortgage, the borrower would pay more than he would with conventional financing.

Given the term of the mortgage (in years), the annual percentage rate, the loan amount, the percentage that the payments increase, and the number of years that the payments increase, the following HP-12C program determines the monthly payments and remaining balance for each year until the level payment is reached.
<table>
<thead>
<tr>
<th>KEYS</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td>00-</td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>01- 43 8</td>
</tr>
<tr>
<td>g END</td>
<td>02- 44 2</td>
</tr>
<tr>
<td>x² y</td>
<td>03- 34</td>
</tr>
<tr>
<td>i</td>
<td>04- 1</td>
</tr>
<tr>
<td>%</td>
<td>05- 25</td>
</tr>
<tr>
<td>1</td>
<td>06- 1</td>
</tr>
<tr>
<td>+</td>
<td>07- 40</td>
</tr>
<tr>
<td>STO 0</td>
<td>08- 44 0</td>
</tr>
<tr>
<td>RCL n</td>
<td>09- 45 11</td>
</tr>
<tr>
<td>RCL 2</td>
<td>10- 45 2</td>
</tr>
<tr>
<td>-</td>
<td>11- 30</td>
</tr>
<tr>
<td>g 12x</td>
<td>12- 43 11</td>
</tr>
<tr>
<td>RCL i</td>
<td>13- 45 12</td>
</tr>
<tr>
<td>g 12+</td>
<td>14- 43 12</td>
</tr>
<tr>
<td>RCL PV</td>
<td>15- 45 13</td>
</tr>
<tr>
<td>STO 3</td>
<td>16- 44 3</td>
</tr>
<tr>
<td>i</td>
<td>17- 1</td>
</tr>
<tr>
<td>CHS</td>
<td>18- 16</td>
</tr>
<tr>
<td>PMT</td>
<td>19- 14</td>
</tr>
<tr>
<td>PV</td>
<td>20- 13</td>
</tr>
<tr>
<td>CHS</td>
<td>21- 16</td>
</tr>
<tr>
<td>FV</td>
<td>22- 15</td>
</tr>
<tr>
<td>i</td>
<td>23- 1</td>
</tr>
<tr>
<td>g 12x</td>
<td>24- 43 11</td>
</tr>
<tr>
<td>RCL PMT</td>
<td>25- 45 14</td>
</tr>
<tr>
<td>RCL 0</td>
<td>26- 45 0</td>
</tr>
<tr>
<td>+</td>
<td>27- 10</td>
</tr>
<tr>
<td>Function</td>
<td>Key(s)</td>
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<tr>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>PMT</td>
<td>28-</td>
</tr>
<tr>
<td>PV</td>
<td>29-</td>
</tr>
<tr>
<td>CHS</td>
<td>30-</td>
</tr>
<tr>
<td>FV</td>
<td>31-</td>
</tr>
<tr>
<td>X</td>
<td>32-</td>
</tr>
<tr>
<td>STO + 1</td>
<td>33-</td>
</tr>
<tr>
<td>RCL 1</td>
<td>34-44</td>
</tr>
<tr>
<td>RCL 2</td>
<td>35-</td>
</tr>
<tr>
<td>-</td>
<td>36-</td>
</tr>
<tr>
<td>g x=0</td>
<td>37-43</td>
</tr>
<tr>
<td>g GTO 40</td>
<td>38-43, 33</td>
</tr>
<tr>
<td>g GTO 25</td>
<td>39-43, 33</td>
</tr>
<tr>
<td>RCL 3</td>
<td>40-</td>
</tr>
<tr>
<td>RCL PV</td>
<td>41-</td>
</tr>
<tr>
<td>+</td>
<td>42-</td>
</tr>
<tr>
<td>STO 4</td>
<td>43-</td>
</tr>
<tr>
<td>RCL 3</td>
<td>44-</td>
</tr>
<tr>
<td>PV</td>
<td>45-</td>
</tr>
<tr>
<td>X</td>
<td>46-</td>
</tr>
<tr>
<td>STO 3</td>
<td>47-</td>
</tr>
<tr>
<td>RCL 3</td>
<td>48-</td>
</tr>
<tr>
<td>R/S</td>
<td>49-</td>
</tr>
<tr>
<td>RCL 4</td>
<td>50-</td>
</tr>
<tr>
<td>X</td>
<td>51-</td>
</tr>
<tr>
<td>RCL 0</td>
<td>52-</td>
</tr>
<tr>
<td>RCL 1</td>
<td>53-</td>
</tr>
<tr>
<td>x^y</td>
<td>54-</td>
</tr>
<tr>
<td>x^y</td>
<td>55-</td>
</tr>
</tbody>
</table>
### Registers

<table>
<thead>
<tr>
<th>n</th>
<th>Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>Used</td>
</tr>
<tr>
<td>FV</td>
<td>Used</td>
</tr>
<tr>
<td>R1</td>
<td>Used</td>
</tr>
<tr>
<td>R2</td>
<td>Used</td>
</tr>
<tr>
<td>R3</td>
<td>Used</td>
</tr>
<tr>
<td>R4</td>
<td>Level Pmt.</td>
</tr>
<tr>
<td>R5-R9</td>
<td>Unused</td>
</tr>
</tbody>
</table>

1. Key in the program.
2. Press \[ \text{CLEAR} \ \text{REG} \].
3. Key in the term of the loan and press \[ n \].
4. Key in the annual interest rate and press \[ i \].
5. Key in the total loan amount and press \[ PV \].
6. Key in the rate of graduation (as a percent) and press \[ \text{ENTER} \].
7. Key in the number of years for which the loan graduates and press \[ \text{R/S} \].
   The following information will be displayed for each year until a level payment is reached.
   a. The current year.
      Then press \[ \text{R/S} \] to continue.
   b. The monthly payment for the current year.
      Then press \[ \text{R/S} \] to continue.
   c. The remaining balance to be paid on the loan at the end of the current year. Then press \[ \text{R/S} \] to return to step a. unless the level payment is reached. If the level payment has been reached, the program will stop, displaying the monthly payment over the remaining term of the loan.
8. For a new case press \[ \text{g GTO} \] 00 and return to step 2.

Example: A young couple recently purchased a new house with a Graduated Payment Mortgage. The loan is for $50,000 over a period of 30 years at an annual interest rate of 12.5%. The monthly payments will be graduating at an annual rate of 5% for the first 5 years and then will be level for the remaining 25 years. What are the monthly payment amount for the first 6 years?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \text{f CLEAR REG} ]</td>
<td>0.00</td>
</tr>
<tr>
<td>30 [ n ]</td>
<td>30.00 Term</td>
</tr>
<tr>
<td>12.5 [ i ]</td>
<td>12.50 Annual interest rate</td>
</tr>
<tr>
<td>50000 [ PV ]</td>
<td>50,000.00 Loan amount</td>
</tr>
<tr>
<td>5 [ \text{ENTER} ]</td>
<td>5.00 Rate of graduation</td>
</tr>
<tr>
<td>5 [ \text{R/S} ]</td>
<td>1.00 Year 1</td>
</tr>
<tr>
<td>[ \text{R/S} ]</td>
<td>-448.88 1st year monthly payment.</td>
</tr>
<tr>
<td>[ \text{R/S} ]</td>
<td>-50,194.67 Remaining balance after 1st year.</td>
</tr>
<tr>
<td>[ \text{R/S} ]</td>
<td>2.00 Year 2</td>
</tr>
</tbody>
</table>
Variable Rate Mortgages

As its name suggests, a variable rate mortgage is a mortgage loan which provides for adjustment of its interest rate as market interest rates change. As a result, the current interest rate on a variable rate mortgage may differ from its origination rate (i.e., the rate when the loan was made). This is the difference between a variable rate mortgage and the standard fixed payment mortgage, where the interest rate and the monthly payment are constant throughout the term.

Under the agreement of the variable rate mortgage, the mortgage is examined periodically to determine any rate adjustments. The rate adjustment may be implemented in two ways:

1. Adjusting the monthly payment.
2. Modifying the term of the mortgage.

The period and limits to interest rate increases vary from state to state. Each periodic adjustment may be calculated by using the HP-12C with the following keystroke procedure. The original terms of the mortgage are assumed to be known.

1. Press \[ g \text{ END} \] and press \[ f \text{ CLEAR FIN} \]
2. Key in the remaining balance of the loan and press \( PV \). The remaining balance is the difference between the loan amount and the total principal from the payments which have been made.

To calculate the remaining balance, do the following:

a. Key in the previous remaining balance. If this is the first mortgage adjustment, this value is the original amount of the loan. Press \( PV \).

b. Key in the annual interest rate before the adjustment (as a percentage) and press \( g \ 12x \).

c. Key in the number of years since the last adjustment. If this is the first mortgage adjustment, then key in the number of years since the origination of the mortgage. Press \( g \ 12x \).

d. Key in the monthly payment over this period and press \( CHS \ PMT \).

e. Press \( FV \) to find the remaining balance, then press \( f \) \( CLEAR \) \( FIN \) \( CHS \) \( PV \).

3. Key in the adjusted annual interest rate (as a percentage) and press \( g \ 12x \). To calculate the new monthly payment:

a. Key in the remaining life of the mortgage (years) and press \( g \ 12x \).

b. Press \( PMT \) to find the new monthly payment.

To calculate the revised remaining term of the mortgage:

c. Key in the present monthly payment and press \( PMT \).

d. Press \( n \ 12 \ 1 \) to find the remaining term of the mortgage in years.

**Example:** A homeowner purchased his house 3 years ago with a $50,000 variable rate mortgage. With a 30-year term, his current monthly payment is $495.15. When the interest rate is adjusted from 11.5% to 11.75%, what will the monthly payment be? If the monthly payment remained unchanged, find the revised remaining term on the mortgage.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g ) END</td>
<td></td>
</tr>
<tr>
<td>( f ) ( CLEAR ) ( FIN ) 50000 ( PV ) ( 11.5 ) ( g ) ( 12x ) 3 ( g ) ( 12x ) 495.15 ( CHS ) ( PMT )</td>
<td>50,000.00 0.96 36.00 -495.15</td>
</tr>
</tbody>
</table>
Skipped Payments

Sometimes a loan (or lease) may be negotiated in which a specific set of monthly payments are going to be skipped each year. Seasonally is usually the reason for such an agreement. For example, because of heavy rainfall, a bulldozer cannot be operated in Oregon during December, January, and February, and the lessee wishes to make payments only when his machinery is being used. He will make nine payments per year, but the interest will continue to accumulate over the months in which a payment is not made.

To find the monthly payment amount necessary to amortize the loan in the specified amount of time, information is entered as follows:

1. Press $g$ END and press $f$ CLEAR FIN.
2. Key in the number of the last payment period before payments close the first time and press $n$.
3. Key in the annual interest rate as a percentage and press $g 12+$ 1 PMT $FV$.
4. Press CHS $PV$ 12 RCL $n - n$ 0 PMT $FV$ STO 0 RCL $n$.
5. Key in the number of payments which are skipped and press $- n 1$ PMT 0 PV $FV$ STO + 0.
6. Press 0 PMT 12 $n 100 PV$ $FV$ RCL $PV$ + CHS $f$ CLEAR FIN $i$.
7. Key in the total number of years in the loan and press $n$.
8. Key in the loan amount and press \( PV \) \( \text{PMT} \) \( RCL \) \( 0 \) \(+\) to obtain the monthly payment amount when the payment is made at the end of the month.

9. Press \( CHS \) \( FV \) \( 0 \) \( \text{PMT} \) \( 1 \) \( n \).

10. Key in the annual interest rate as a percent and press \( g \) \( 12\div \) \( PV \) to find the monthly payment amount when the payment is made at the beginning of the month.

**Example:** A bulldozer worth $100,000 is being purchased in September. The first payment is due one month later, and payments will continue over a period of 5 years. Due to the weather, the machinery will not be used during the winter months, and the purchaser does not wish to make payments during January, February, and March (months 4 thru 6). If the current interest rate is 14%, what is the monthly payment necessary to amortize the loan?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g ) ( \text{END} )</td>
<td>3.00</td>
</tr>
<tr>
<td>( f ) ( \text{CLEAR} ) ( \text{FIN} ) ( 3 ) ( n )</td>
<td></td>
</tr>
<tr>
<td>14 ( g ) ( 12\div )</td>
<td></td>
</tr>
<tr>
<td>1 ( \text{PMT} ) ( FV ) ( CHS ) ( PV )</td>
<td>3,119.98</td>
</tr>
<tr>
<td>12 ( RCL ) ( n ) ( - ) ( n )</td>
<td></td>
</tr>
<tr>
<td>0 ( \text{PMT} ) ( FV ) ( \text{STO} ) ( 0 )</td>
<td></td>
</tr>
<tr>
<td>RCL ( n )</td>
<td></td>
</tr>
<tr>
<td>3 ( - ) ( n )</td>
<td></td>
</tr>
<tr>
<td>1 ( \text{PMT} )</td>
<td></td>
</tr>
<tr>
<td>0 ( PV ) ( FV )</td>
<td></td>
</tr>
<tr>
<td>( \text{STO} ) ( + ) ( 0 )</td>
<td></td>
</tr>
<tr>
<td>0 ( \text{PMT} )</td>
<td></td>
</tr>
<tr>
<td>12 ( n )</td>
<td></td>
</tr>
<tr>
<td>100 ( PV ) ( FV )</td>
<td></td>
</tr>
<tr>
<td>RCL ( PV ) ( + ) ( CHS )</td>
<td></td>
</tr>
<tr>
<td>( f ) ( \text{CLEAR} ) ( \text{FIN} ) ( i )</td>
<td></td>
</tr>
<tr>
<td>5 ( n )</td>
<td></td>
</tr>
<tr>
<td>100000 ( PV )</td>
<td></td>
</tr>
<tr>
<td>( \text{PMT} ) ( RCL ) ( 0 ) (+).</td>
<td></td>
</tr>
</tbody>
</table>
Savings

Initial Deposit with Periodic Deposits

Given an initial deposit into a savings account, and a series of periodic deposits coincident with the compounding period, the future value (or accumulated amount) may be calculated as follows:

1. Press \( \text{g} \) \( \text{END} \) and press \( \text{f} \) \( \text{CLEAR} \) \( \text{FIN} \).
2. Key in the initial investment and press \( \text{CHS} \) \( \text{PV} \).
3. Key in the number of additional periodic deposits and press \( n \).
4. Key in the periodic interest rate and press \( i \).
5. Key in the periodic deposit and press \( \text{CHS} \) \( \text{PMT} \).
6. Press \( \text{FV} \) to determine the value of the account at the end of the time period.

**Example:** You have just opened a savings account with a $200 deposit. If you deposit $50 a month, and the account earns 5 1/4% compounded monthly, how much will you have in 3 years?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{g} ) ( \text{END} )</td>
<td></td>
</tr>
<tr>
<td>( \text{f} ) ( \text{CLEAR} ) ( \text{FIN} )</td>
<td></td>
</tr>
<tr>
<td>200 ( \text{CHS} ) ( \text{PV} )</td>
<td></td>
</tr>
<tr>
<td>3 ( \text{g} ) ( 12 \times )</td>
<td></td>
</tr>
<tr>
<td>5.25 ( \text{g} ) ( 12+ )</td>
<td></td>
</tr>
<tr>
<td>50 ( \text{CHS} ) ( \text{PMT} ) ( \text{FV} )</td>
<td>2,178.94</td>
</tr>
</tbody>
</table>

**Value of the account.**

**Note:** If the periodic deposits do not coincide with the compounding periods, the account must be evaluated in another manner. First, find the future value of the initial deposits and store it. Then use the procedure for compounding periods different from payment periods to calculate the future value of the periodic deposits. Recall the future value of the initial deposit and add to obtain the value of the account.
Number of Periods to Deplete a Savings Account or to Reach a Specified Balance.

Given the current value of a savings account, the periodic interest rate, the amount of the periodic withdrawal, and a specified balance, this procedure determines the number of periods to reach that balance (the balance is zero if the account is depleted).

1. Press \( \text{g} \) END and press \( \text{f} \) CLEAR FIN.
2. Key in the value of the savings account and press CHS PV.
3. Key in the periodic interest rate and press \( \text{i} \).
4. Key in the amount of the periodic withdrawal and press PMT.
5. Key in the amount remaining in the account and press FV. This step may be omitted if the account is depleted (FV=0).
6. Press \( \text{n} \) to determine the number of periods to reach the specified balance.

Example: Your savings account presently contains $18,000 and earns 5 1/4% compounded monthly. You wish to withdraw $300 a month until the account is depleted. How long will this take? If you wish to reduce the account to $5,000, how many withdrawals can you make?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{g} ) END</td>
<td></td>
</tr>
<tr>
<td>( \text{f} ) CLEAR FIN</td>
<td>71.00 Months to deplete account.</td>
</tr>
<tr>
<td>18000 CHS PV</td>
<td></td>
</tr>
<tr>
<td>5.25 ( \text{g} ) 12+</td>
<td>53.00 Months to reduce the account to $5,000</td>
</tr>
<tr>
<td>300 PMT n</td>
<td></td>
</tr>
<tr>
<td>5000 FV n</td>
<td></td>
</tr>
</tbody>
</table>

Periodic Deposits and Withdrawals

This section is presented as a guideline for evaluating a savings plan when deposits and withdrawals occur at irregular intervals. One problem is given, and a step by step method for setting up and solving the problem is presented:

Example: You are presently depositing $50 and the end of each month into a local savings and loan, earning 5 1/2% compounded monthly. Your current balance is $1023.25. How much will you have accumulated in 5 months?
The cash flow diagram looks like this:

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 \\
-50 & -50 & -50 & -50 & -50 \\
\end{array}
\]

\[PV = -1023.25\]

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g) END</td>
<td></td>
</tr>
<tr>
<td>(f) CLEAR, FIN</td>
<td>1,299.22 Amount in account.</td>
</tr>
<tr>
<td>50 CHS, PMT</td>
<td>1,299.22</td>
</tr>
<tr>
<td>5.5 (g) 12 ÷</td>
<td></td>
</tr>
<tr>
<td>1023.25 CHS, PV</td>
<td></td>
</tr>
<tr>
<td>5 (n) FV</td>
<td></td>
</tr>
</tbody>
</table>

Now suppose that at the beginning of the 6th month you withdrew $80. What is the new balance?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>80 -</td>
<td>1,219.22 New balance.</td>
</tr>
</tbody>
</table>

You increase your monthly deposit to $65. How much will you have in 3 months?

The cash flow diagram looks like this:
Suppose that for 2 months you decide not to make a periodic deposit. What is the balance in the account?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
</table>

This type of procedure may be continued for any length of time, and may be modified to meet the user’s particular needs.

**Savings Account Compounded Daily**

This HP 12C program determines the value of a savings account when interest is compounded daily, based on a 365 day year. The user is able to
calculate the total amount remaining in the account after a series of transactions on specified dates.

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>P/R</td>
</tr>
<tr>
<td>f</td>
<td>CLEAR PRGM</td>
</tr>
<tr>
<td>CHS</td>
<td>02- 13</td>
</tr>
<tr>
<td>PV</td>
<td>03- 33</td>
</tr>
<tr>
<td>R↓</td>
<td>04- 3</td>
</tr>
<tr>
<td>5</td>
<td>05- 6</td>
</tr>
<tr>
<td>5</td>
<td>06- 5</td>
</tr>
<tr>
<td>+</td>
<td>07- 10</td>
</tr>
<tr>
<td>1</td>
<td>08- 12</td>
</tr>
<tr>
<td>R↓</td>
<td>09- 33</td>
</tr>
<tr>
<td>STO 0</td>
<td>10- 44 0</td>
</tr>
<tr>
<td>RCL PV</td>
<td>11- 15 13</td>
</tr>
<tr>
<td>CHS</td>
<td>12- 16</td>
</tr>
<tr>
<td>R/S</td>
<td>13- 31</td>
</tr>
<tr>
<td>STO 2</td>
<td>14- 44 2</td>
</tr>
<tr>
<td>R↓</td>
<td>15- 33</td>
</tr>
<tr>
<td>STO 1</td>
<td>16- 44 1</td>
</tr>
<tr>
<td>RCL 0</td>
<td>17- 45 0</td>
</tr>
<tr>
<td>RCL 1</td>
<td>18- 45 1</td>
</tr>
<tr>
<td>g</td>
<td>ADYS</td>
</tr>
<tr>
<td>n</td>
<td>20- 11</td>
</tr>
<tr>
<td>FV</td>
<td>21- 15</td>
</tr>
<tr>
<td>f</td>
<td>RND</td>
</tr>
<tr>
<td>FV</td>
<td>23- 15</td>
</tr>
<tr>
<td>ENTER</td>
<td>24- 36</td>
</tr>
<tr>
<td>RCL PV</td>
<td>25- 45 13</td>
</tr>
</tbody>
</table>
1. Key in the program
2. Press $\text{f} \ \text{CLEAR} \ \text{REG}$ and press $g \ M \ D Y$.
3. Key in the date (MM.DDYYYY) of the first transaction and press $\text{ENTER}$.
4. Key in the annual nominal interest rate as a percentage and press $\text{ENTER}$.
5. Key in the amount of the initial deposit and press $R/S$.
6. Key in the date of the next transaction and press $\text{ENTER}$.
7. Key in the amount of the transaction (positive for money deposited, negative for cash withdrawn) and press $R/S$ to determine the amount in the account.
8. Repeat steps 6 and 7 for subsequent transactions.
9. To see the total interest to date, press $\text{RCL} \ 3$. 

<table>
<thead>
<tr>
<th>+</th>
<th>26 - 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{STO} \ + \ 3$</td>
<td>27-44 40 3</td>
</tr>
<tr>
<td>$\text{RCL} \ FV$</td>
<td>28-45 15</td>
</tr>
<tr>
<td>$\text{RCL} \ 2$</td>
<td>29-45 2</td>
</tr>
<tr>
<td>+</td>
<td>30-40</td>
</tr>
<tr>
<td>$\text{CHS}$</td>
<td>31-16</td>
</tr>
<tr>
<td>$\text{PV}$</td>
<td>32-13</td>
</tr>
<tr>
<td>$\text{RCL} \ 1$</td>
<td>33-45 1</td>
</tr>
<tr>
<td>$\text{STO} \ 0$</td>
<td>34-44 0</td>
</tr>
<tr>
<td>$\text{RCL} \ PV$</td>
<td>35-45 13</td>
</tr>
<tr>
<td>$\text{CHS}$</td>
<td>36-16</td>
</tr>
<tr>
<td>$g \ \text{GTO} \ 13$</td>
<td>37-43, 33 13</td>
</tr>
<tr>
<td>$\text{f} \ \text{P/R}$</td>
<td></td>
</tr>
</tbody>
</table>

**REGISTERS**

- $\Delta$days
- $l$: $l/365$
- $\text{PV}$: Used
- $\text{PMT}$: 0
- $\text{FV}$: Used
- $R_0$: Initial date
- $R_1$: Next date
- $R_2$: $\text{}$ amount
- $R_3$: Interest
- $R_4$-$R_{14}$: Unused

43
10. For a new case press [f][PRGM] and go to step 2.

**Example:** Compute the amount remaining in this 5.25% account after the following transactions:

1. January 19, 1981 deposit $125.00
2. February 24, 1981 deposit $60.00
3. March 16, 1981 deposit $70.00
4. April 6, 1981 withdraw $50.00
5. June 1, 1981 deposit $175.00
6. July 6, 1981 withdraw $100.00

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>[f] CLEAR REG</td>
<td></td>
</tr>
<tr>
<td>g M.DY</td>
<td></td>
</tr>
<tr>
<td>1.191981 ENTER</td>
<td>125.00</td>
</tr>
<tr>
<td>5.25 ENTER</td>
<td>Initial Deposit.</td>
</tr>
<tr>
<td>125 R/S</td>
<td></td>
</tr>
<tr>
<td>2.241981 ENTER</td>
<td>185.65</td>
</tr>
<tr>
<td>60 R/S</td>
<td>Balance in account, February 24, 1981.</td>
</tr>
<tr>
<td>3.161981 ENTER</td>
<td>256.18</td>
</tr>
<tr>
<td>70 R/S</td>
<td>Balance in account, March 16, 1981.</td>
</tr>
<tr>
<td>4.061981 ENTER</td>
<td>206.95</td>
</tr>
<tr>
<td>50 R/S</td>
<td>Balance in account, April 6, 1981.</td>
</tr>
<tr>
<td>6.0111981 ENTER</td>
<td>383.62</td>
</tr>
<tr>
<td>175 R/S</td>
<td>Balance in account, June 1, 1981.</td>
</tr>
<tr>
<td>7.061981 ENTER</td>
<td>285.56</td>
</tr>
<tr>
<td>100 R/S</td>
<td>Balance in account, July 6, 1981.</td>
</tr>
<tr>
<td>RCL 3 R/S</td>
<td>5.56</td>
</tr>
<tr>
<td></td>
<td>Total interest.</td>
</tr>
</tbody>
</table>

**Compounding Periods Different From Payment Periods**

In financial calculations involving a series of payments equally spaced in time with periodic compounding, both periods of time are normally equal and coincident. This assumption is preprogrammed into the HP 12C.
I savings plans however, money may become available for deposit or investment at a frequency different from the compounding frequencies offered. The HP 12C can easily be used in these calculations. However, because of the assumptions mentioned the periodic interest rate must be adjusted to correspond to an equivalent rate for the payment period.

Payments deposited for a partial compounding period will accrue simple interest for the remainder of the compounding period. This is often the case, but may not be true for all institutions.

These procedures present solutions for future value, payment amount, and number of payments. In addition, it should be noted that only annuity due (payments at the beginning of payment period) calculations are shown since this is the most common in savings plan calculations.

To calculate the equivalent payment period interest rate, information is entered as follows:

1. Press \( \text{g} \) \( \text{REG} \) and press \( \text{f} \) \( \text{CLEAR} \) \( \text{FIN} \).
2. Key in the annual interest rate (as a percent) and press \( \text{ENTER} \).
3. Key in the number of compounding periods per year and press \( \text{n} \) + \( \text{i} \).
4. Key in 100 and press \( \text{PV} \) \( \text{FV} \).
5. Key in the number of payments (deposits) per year and press \( \text{n} \) \( \text{i} \) \( \text{f} \) \( \text{CLEAR} \) \( \text{FIN} \) \( \text{i} \).

The interest rate which corresponds to the payment period is now in register "i" and you are ready to proceed.

**Example 1: Solving for future value.**

Starting today you make monthly deposits of $25 into an account paying 5% compounded daily (365-day basis). At the end of 7 years, how much will you receive from the account?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{g} ) ( \text{BEG} ) ( \text{f} ) ( \text{CLEAR} ) ( \text{FIN} ) ( \text{enter} ) ( 365 ) ( \text{n} ) + ( \text{i} ) ( 12 ) ( \text{n} ) ( \text{i} ) ( \text{f} ) ( \text{CLEAR} ) ( \text{FIN} ) ( \text{i} )</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Equivalent periodic interest rate.

45
Example 2: Solving for payment amount.
For 8 years you wish to make weekly deposits in a savings account paying 5.5% compounded quarterly. What amount must you deposit each week to accumulate $6000.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7 \ \text{g} \ \text{12x}) (25 \ \text{CHS} \ \text{PMT} \ \text{FV})</td>
<td>2,519.61 Future value.</td>
</tr>
</tbody>
</table>

\text{Example 3: Solving for number of payment periods.}
You can make weekly deposits of $10 in to an account paying 5.25% compounded daily (365-day basis). How long will it take you to accumulate $1000?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9 \ \text{BEG} \ \text{f CLEAR} \ \text{FIN} \ 5.5 \ \text{ENTER}) (4 \ \text{n} \ \div \ \text{i}) (100 \ \text{FV} \ \text{FV}) (52 \ \text{n} \ \text{i} \ \text{f CLEAR} \ \text{FIN} \ \text{i})</td>
<td>0.11 Equivalent periodic interest rate.</td>
</tr>
<tr>
<td>8 \ \text{ENTER} \ 52 \ \times \ \text{n} \ \text{6000} \ \text{FV} \ \text{PMT})</td>
<td>-11.49 Periodic payment.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9 \ \text{BEG} \ \text{f CLEAR} \ \text{FIN} \ 5.25 \ \text{ENTER}) (365 \ \text{n} \ \div \ \text{i}) (100 \ \text{PV} \ \text{FV}) (52 \ \text{n} \ \text{i} \ \text{f CLEAR} \ \text{FIN} \ \text{i})</td>
<td>0.10 Equivalent periodic interest rate.</td>
</tr>
<tr>
<td>8 \ \text{CHS} \ \text{PMT} \ \text{96.00} \ \text{FV} \ \text{n})</td>
<td>96.00 Weeks.</td>
</tr>
</tbody>
</table>
Investment Analysis

Lease vs. Purchase

An investment decision frequently encountered is the decision to lease or purchase capital equipment or buildings. Although a thorough evaluation of a complex acquisition usually requires the services of a qualified accountant, it is possible to simplify a number of the assumptions to produce a first approximation.

The following HP-12C program assumes that the purchase is financed with a loan and that the loan is made for the term of the lease. The tax advantages of interest paid, depreciation, and the investment credit which accrues from ownership are compared to the tax advantage of treating the lease payment as an expense. The resulting cash flows are discounted to the present at the firm's after-tax cost of capital.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td>00-</td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>01- 30</td>
</tr>
<tr>
<td>RCL 1</td>
<td>02- 1</td>
</tr>
<tr>
<td>+ 0</td>
<td>03-44 40 0</td>
</tr>
<tr>
<td>STO 8</td>
<td>04- 45 3</td>
</tr>
<tr>
<td>RCL 3</td>
<td>05- 30</td>
</tr>
<tr>
<td>-</td>
<td>06- 20</td>
</tr>
<tr>
<td>X</td>
<td>07- 44 8</td>
</tr>
<tr>
<td>STO 1</td>
<td>08- 1</td>
</tr>
<tr>
<td>f AMORT</td>
<td>09- 42 11</td>
</tr>
<tr>
<td>STO 9</td>
<td>10- 44 1</td>
</tr>
<tr>
<td>RCL PV</td>
<td>11- 45 13</td>
</tr>
<tr>
<td>STO 9</td>
<td>12- 44 9</td>
</tr>
<tr>
<td>RCL PMT</td>
<td>13- 45 14</td>
</tr>
<tr>
<td>STO 9</td>
<td>14-44 48 0</td>
</tr>
</tbody>
</table>

47
<table>
<thead>
<tr>
<th>Command</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCL n</td>
<td>15 - 45 11</td>
</tr>
<tr>
<td>STO · 1</td>
<td>16 - 44 48 1</td>
</tr>
<tr>
<td>RCL i</td>
<td>17 - 45 12</td>
</tr>
<tr>
<td>STO · 2</td>
<td>18 - 44 48 2</td>
</tr>
<tr>
<td>RCL 5</td>
<td>19 - 45 5</td>
</tr>
<tr>
<td>PV</td>
<td>20 - 13</td>
</tr>
<tr>
<td>RCL 6</td>
<td>21 - 45 6</td>
</tr>
<tr>
<td>n</td>
<td>22 - 11</td>
</tr>
<tr>
<td>RCL 7</td>
<td>23 - 45 7</td>
</tr>
<tr>
<td>i</td>
<td>24 - 12</td>
</tr>
<tr>
<td>RCL 0</td>
<td>25 - 45 0</td>
</tr>
<tr>
<td>f SOYD</td>
<td>26 - 42 24</td>
</tr>
<tr>
<td>STO + 1</td>
<td>27 - 44 40 1</td>
</tr>
<tr>
<td>RCL 9</td>
<td>28 - 45 9</td>
</tr>
<tr>
<td>PV</td>
<td>29 - 13</td>
</tr>
<tr>
<td>RCL . 0</td>
<td>30 - 45 48 0</td>
</tr>
<tr>
<td>PMT</td>
<td>31 - 14</td>
</tr>
<tr>
<td>RCL . 1</td>
<td>32 - 45 48 1</td>
</tr>
<tr>
<td>n</td>
<td>33 - 11</td>
</tr>
<tr>
<td>RCL . 2</td>
<td>34 - 45 48 2</td>
</tr>
<tr>
<td>i</td>
<td>35 - 12</td>
</tr>
<tr>
<td>RCL 1</td>
<td>36 - 45 1</td>
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<tr>
<td>RCL 3</td>
<td>37 - 45 3</td>
</tr>
<tr>
<td>x</td>
<td>38 - 20</td>
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<tr>
<td>RCL PMT</td>
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</tr>
<tr>
<td>-</td>
<td>40 - 30</td>
</tr>
<tr>
<td>RCL 8</td>
<td>41 - 45 8</td>
</tr>
<tr>
<td>-</td>
<td>42 - 30</td>
</tr>
</tbody>
</table>
Instructions:
1. Key in the program.
   - Select the depreciation function and key in at line 26.
2. Press \( \text{g} \) [END] and press \( \text{f} \) [CLEAR] [REG].
3. Input the following information for the purchase of the loan:
   - Key in the number of years for amortization and press \( \text{n} \).
   - Key in the annual interest rate and press \( \text{i} \).
   - Key in the loan amount (purchase price) and press \( \text{CHS} \) [PV].
   - Press \( \text{PMT} \) to find the annual payment.
4. Key in the marginal effective tax rate (as a decimal) and press \( \text{STO} \) 3.
5. Key in the discount rate (as a decimal) or cost of capital and press \( \text{ENTER} \) \( \text{1} \) \( \text{+} \) \( \text{STO} \) 4.
6. Key in the depreciable value and press \( \text{STO} \) 5.
7. Key in the depreciable live and press \( \text{STO} \) 6.
8. For declining balance depreciation, key in the depreciation factor (as a percentage) and press \texttt{STO} 7.

9. Key in the total first lease payment (including any advance payments) and press \texttt{ENTER} 1 \texttt{RCL} 3 \texttt{–} \texttt{x} \texttt{STO} 2.

10. Key in the first year’s maintenance expense that would be anticipated if the asset was owned and press . If the lease contract does not include maintenance, then it is not a factor in the lease vs. purchase decision and 0 expense should be used.

11. Key in the next lease payment and press \texttt{R/S}. During any year in which a lease payment does not occur (e.g. the last several payments of an advance payment contract) use 0 for the payment.

12. Repeat steps 10 and 11 for all maintenance expenses and lease payments over the term of the analysis. Optional - If the investment tax credit is taken, key in the amount of the credit after finishing steps 10 and 11 for the year in which the credit is taken and press \texttt{g} \texttt{GTO} 43 \texttt{R/S}; Continue steps 10 and 11 for the remainder of the term.

13. After all the lease payments and expenses have been entered (steps 10 and 11), key in the lease buy back option and press \texttt{g} \texttt{GTO} 43 \texttt{R/S}; If no buy back option exists, use the estimated salvage value of the purchased equipment at the end of the term.

14. To find the net advantage of owning press \texttt{RCL} 2. A negative value represents a net lease advantage.

\textbf{Example:} Home Style Bagel Company is evaluating the acquisition of a mixer which can be leased for $1700 a year with the first and last payments in advance and a $750 buy back option at the end of 10 years (maintenance is included). The same equipment could be purchased for $10,000 with a 12% loan amortized over 10 years. Ownership maintenance is estimated to be 2% of the purchase price per year for the first for years. A major overhaul is predicted for the 5th year at a cost of $1500. Subsequent yearly maintenance of 3% is estimated for the remainder of the 10-year term. The company would use sum of the years digits depreciation on a 10 year life with $1500 salvage value. An accountant informs management to take the 10% capital investment tax credit at the end of the second year and to figure the cash flows at a 48% tax rate. The after tax cost of capital (discounting rate) is 5 percent.

Because lease payments are made in advance and standard loan payments are made in arrears the following cash flow schedule is appropriate for a lease with the last payment in advance.

<table>
<thead>
<tr>
<th>Year</th>
<th>Maintenance</th>
<th>Lease Payment</th>
<th>Tax Credit</th>
<th>Buy Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1700+700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>200</td>
<td>1700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Keystrokes | Display
--- | ---
\texttt{\textbf{g}} \texttt{BEG} | 0.00
\texttt{\textbf{f}} \texttt{CLEAR} \texttt{REG} | 
10 \texttt{\textbf{n}} 12 \texttt{i} | -10,000.00 Always use negative loan amount.
10000 \texttt{CHS PV} | 
PMT | 1,769.84 Purchase payment.
\texttt{.48 STO 3} | 0.48 Marginal tax rate.
\texttt{.05 \textbf{ENTER}} | 1.05 Discounting factor.
1 \texttt{+ STO 4} | 
10000 \texttt{ENTER} | 8,500.00 Depreciable value.
1500 \texttt{STO 5} | 
10 \texttt{STO 6} | 10.00 Depreciable life.
1700 \texttt{ENTER \textbf{+}} | 3,400.00 1st lease payment.
1 \texttt{RCL 3 \textbf{\textbf{\textbf{\textbf{-}}}}} \texttt{x STO 2} | 1,768.00 After-tax expense.
200 \texttt{ENTER} | 312.36 Present value of 1st year's net purchase.
1700 \texttt{R/S} | 
200 \texttt{ENTER} | 200.43 2nd year's advantage.
1700 \texttt{R/S} | 
1000 \texttt{g GTO 43} | 1,000.00 Tax credit.
\texttt{R/S} | 907.03 Present value of tax credit.
200 \texttt{ENTER} | 95.05 3rd year.
1700 \texttt{R/S} | 
200 \texttt{ENTER} | -4.38 4th year.
1700 \texttt{R/S} | 

51
Break-Even Analysis

Break-even analysis is basically a technique for analyzing the relationships among fixed costs, variable costs, and income. Until the break even point is reached at the intersection of the total income and total cost lines, the producer operates at a loss. After the break-even point each unit produced and sold makes a profit. Break even analysis may be represented as follows.

<table>
<thead>
<tr>
<th>Year</th>
<th>Income</th>
<th>Outflow</th>
<th>Net</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th</td>
<td>200</td>
<td>-628.09</td>
<td></td>
<td>-628.09</td>
</tr>
<tr>
<td>6th</td>
<td>200</td>
<td>-226.44</td>
<td></td>
<td>-226.44</td>
</tr>
<tr>
<td>7th</td>
<td>200</td>
<td>-309.48</td>
<td></td>
<td>-309.48</td>
</tr>
<tr>
<td>8th</td>
<td>200</td>
<td>-388.81</td>
<td></td>
<td>-388.81</td>
</tr>
<tr>
<td>9th</td>
<td>300</td>
<td>-1,034.72</td>
<td></td>
<td>-1,034.72</td>
</tr>
<tr>
<td>10th</td>
<td>300</td>
<td>-1,080.88</td>
<td></td>
<td>-1,080.88</td>
</tr>
</tbody>
</table>

Buy back: 750.00

After tax buy back expense: 390.00

Present value: 239.43

Net lease advantage: -150.49

52
The variables are: fixed costs \((F)\), Sales price per unit \((P)\), variable cost per unit \((V)\), number of units sold \((U)\), and gross profit \((GP)\). One can readily evaluate \(GP, U \text{ or } P\) given the four other variables. To calculate the break-even volume, simply let the gross profit equal zero and calculate the number of units sold \((U)\).

To calculate the break-even volume:

1. Key in the fixed costs and press \(\text{ENTER}\).
2. Key in the unit price and press \(\text{ENTER}\).
3. Key in the variable cost per unit and press \(\text{X}\).
4. Press \(+\) to calculate the break-even volume.

To calculate the gross profit at a given volume:

1. Key in the unit price and press \(\text{ENTER}\).
2. Key in the variable cost per unit and press \(\text{X}\).
3. Key in the number of units sold and press \(\text{X}\).
4. Key in the fixed cost and press \(\text{X}\) to calculate the gross profit.
To calculate the sales volume needed to achieve a specified gross profit:

1. Key in the desired gross profit and press \( \text{ENTER} \).
2. Key in the fixed cost and press \( \text{+} \).
3. Key in sales price per unit and press \( \text{ENTER} \).
4. Key in the variable cost per unit and press \( \text{—} \).
5. Press \( \text{÷} \) to calculate the sales volume.

To calculate the required sales price to achieve a given gross profit at a specified sales volume:

1. Key in the fixed costs and press \( \text{ENTER} \).
2. Key in the gross desired and press \( \text{ENTER} \).
3. Key in the specified sales volume in units and press \( \text{÷} \).
4. Key in the variable cost per unit and press \( \text{+} \) to calculate the required sales price per unit.

**Example 1:** The E.Z. Sells company markets textbooks on salesmanship. The fixed cost involved in setting up to print the books are $12,000. The variable cost per copy, including printing and marketing the books are $6.75 per copy. The sales price per copy is $13.00. How many copies must be sold to break even?

Find the gross profit if 2500 units are sold.

If a gross profit of $4,500 is desired at a sales volume of 2500 units, what should the sales price be?
For repeated calculation the following HP-12C program can be used.

<table>
<thead>
<tr>
<th>KEYS</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>12000</code></td>
<td><code>12,000.00</code></td>
</tr>
<tr>
<td><code>+</code></td>
<td><code>16,500.00</code></td>
</tr>
<tr>
<td><code>+</code></td>
<td><code>6.60</code></td>
</tr>
<tr>
<td><code>+</code></td>
<td><code>13.35</code></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KEYS</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>f</code></td>
<td><code>[P/R]</code></td>
</tr>
<tr>
<td><code>f</code></td>
<td><code>[CLEAR]</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>3</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>2</code></td>
</tr>
<tr>
<td><code>-</code></td>
<td><code>03-</code></td>
</tr>
<tr>
<td><code>g</code></td>
<td><code>[GTO]</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>4</code></td>
</tr>
<tr>
<td><code>x</code></td>
<td><code>06-</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>1</code></td>
</tr>
<tr>
<td><code>-</code></td>
<td><code>08-</code></td>
</tr>
<tr>
<td><code>g</code></td>
<td><code>[GTO]</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>5</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>1</code></td>
</tr>
<tr>
<td><code>+</code></td>
<td><code>12-</code></td>
</tr>
<tr>
<td><code>x</code></td>
<td><code>y</code></td>
</tr>
<tr>
<td><code>+</code></td>
<td><code>14-</code></td>
</tr>
<tr>
<td><code>g</code></td>
<td><code>[GTO]</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>1</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>5</code></td>
</tr>
<tr>
<td><code>+</code></td>
<td><code>18-</code></td>
</tr>
<tr>
<td><code>RCL</code></td>
<td><code>4</code></td>
</tr>
<tr>
<td><code>+</code></td>
<td><code>20-</code></td>
</tr>
</tbody>
</table>
1. Key in the program and store the know variables as follows:
   a. Key in the fixed costs, \( F \) and press \( \text{STO} \ 1 \).
   b. Key in the variable costs per unit, \( V \) and press \( \text{STO} \ 2 \).
   c. Key in the unit price, \( P \) (if known) and press \( \text{STO} \ 3 \).
   d. Key in the sales volume, \( U \), in units (if known) and press \( \text{STO} \ 4 \).
   e. Key in the gross profit, \( GP \), (if known) and press \( \text{STO} \ 5 \).

2. To calculate the sales volume to achieve a desired gross profit:
   a. Store values as shown in 1a, 1b, and 1c.
   b. Key in the desired gross profit (zero for break even) and press \( \text{STO} \ 5 \).
   c. Press \( \text{R/S} \ \text{R/S} \ \text{R/S} \ \text{g} \ \text{GTO} \ 10 \ \text{R/S} \) to calculate the required volume.

3. To calculate the gross profit at a given sales volume.
   a. Store values as shown in 1a, 1b, 1c, and 1d.
   b. Press \( \text{R/S} \ \text{g} \ \text{GTO} \ 05 \ \text{R/S} \) to calculate gross profit.

4. To calculate the sales price per unit to achieve a desired gross profit at a specified sales volume:
   a. Store values as shown in 1a, 1b, 1d, and 1e.
   b. Press \( \text{g} \ \text{GTO} \ 16 \ \text{R/S} \) to calculate the required sales price.
Example 2: A manufacturer of automotive accessories produces rear view mirrors. A new line of mirrors will require fixed costs of $35,000 to produce. Each mirror has a variable cost of $8.25. The price of mirrors is tentatively set at $12.50 each. What volume is needed to break even?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>35000 STO 1</td>
<td>35,000.00 Fixed cost.</td>
</tr>
<tr>
<td>8.25 STO 2</td>
<td>8.25 Variable cost.</td>
</tr>
<tr>
<td>12.5 STO 3</td>
<td>12.50 Sales price.</td>
</tr>
<tr>
<td>0 STO 5</td>
<td>0.00</td>
</tr>
<tr>
<td>R/S 8 GTO 10 R/S</td>
<td>8,235.29 Break-even volume is between 8,235 and 8,236 units.</td>
</tr>
</tbody>
</table>

What would be the gross profit if the price is raised to $14.00 and the sales volume is 10,000 units?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 STO 3</td>
<td>14.00 Sales price.</td>
</tr>
<tr>
<td>10000 STO 4</td>
<td>10,000.00 Volume.</td>
</tr>
<tr>
<td>R/S 8 GTO 5 R/S</td>
<td>22,500.00 Gross Profit.</td>
</tr>
</tbody>
</table>

Operating Leverage

The degree of operating leverage (OL) at a point is defined as the ratio of the percentage change in net operating income to the percentage change in units sold. The greatest degree of operating leverage is found near the break even point where a small change in sales may produce a very large increase in profits. Likewise, firms with a small degree of operating leverage are operating farther from the break even point, and they are relatively insensitive to changes in sales volume.

The necessary inputs to calculate the degree of operating leverage and fixed costs ($F$), sales price per unit ($P$), variable cost per unit ($V$) and number of units ($U$).

The operating leverage may be readily calculated as follows:

1. Key in the sales price per unit and press ENTER.
2. Key in the variable cost per unit and press   .
3. Key in the number of units and press \[x\] ENTER ENTER.

4. Key in the fixed cost and press \[-\] \[\div\] to obtain the operating leverage.

**Example 1:** For the data given in example 1 of the Break-Even Analysis section, calculate the operating leverage at 2000 units and at 5000 units when the sales price is $13 a copy.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 ENTER</td>
<td>13.00</td>
</tr>
<tr>
<td>6.75 (-)</td>
<td>6.25</td>
</tr>
<tr>
<td>2000 [x] ENTER</td>
<td>25.00</td>
</tr>
<tr>
<td>ENTER 12000 [-] [\div]</td>
<td></td>
</tr>
<tr>
<td>13 ENTER</td>
<td>13.00</td>
</tr>
<tr>
<td>6.75 (-)</td>
<td>6.25</td>
</tr>
<tr>
<td>5000 [x] ENTER</td>
<td>1.62</td>
</tr>
<tr>
<td>ENTER 12000 [-] [\div]</td>
<td></td>
</tr>
</tbody>
</table>

For repeated calculations the following HP-12C program can be used:

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>[f] P/R</td>
<td>00-</td>
</tr>
<tr>
<td>[f] CLEAR PRGM</td>
<td>01- 45 3</td>
</tr>
<tr>
<td>RCL 3</td>
<td>02- 45 2</td>
</tr>
<tr>
<td>RCL 2</td>
<td>03- 30</td>
</tr>
<tr>
<td>(-)</td>
<td>04- 20</td>
</tr>
<tr>
<td>[x]</td>
<td>05- 36</td>
</tr>
<tr>
<td>ENTER</td>
<td>06- 36</td>
</tr>
<tr>
<td>ENTER</td>
<td>07- 45 1</td>
</tr>
<tr>
<td>RCL 1</td>
<td>08- 30</td>
</tr>
<tr>
<td>(-)</td>
<td>09- 10</td>
</tr>
<tr>
<td>[g] GTO 00</td>
<td>10-43, 33 00</td>
</tr>
<tr>
<td>[f] P/R</td>
<td></td>
</tr>
</tbody>
</table>
1. Key in the program.
2. Key in and store input variables $F$, $V$, and $P$ as described in the Break-Even Analysis program.
3. Key in the sales volume and press $R/S$ to calculate the operating leverage.
4. To calculate a new operating leverage at a different sales volume, key in the new sales volume and press $R/S$.

Example 2: For the figures given in example 2 of the Break-Even Analysis section, calculate the operating leverage at a sales volume of 9,000 and 20,000 units if the sales price is $12.50 per unit.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>35000</td>
<td>STO 1</td>
</tr>
<tr>
<td>8.25</td>
<td>STO 2</td>
</tr>
<tr>
<td>12.5</td>
<td>STO 3</td>
</tr>
<tr>
<td>9000</td>
<td>R/S</td>
</tr>
<tr>
<td>20000</td>
<td>R/S</td>
</tr>
</tbody>
</table>

**Profit and Loss Analysis**

The HP-12C may be programmed to perform simplified profit and loss analysis using the standard profit income formula and can be used as a dynamic simulator to quickly explore ranges of variables affecting the profitability of a marketing operation.

The program operates with net income return and operating expenses as percentages. Both percentage figures are based on net sales price.

It may also be used to simulate a company wide income statement by replacing list price with gross sales and manufacturing cost with cost of goods sold.
Any of the five variables: a) list price, b) discount (as a percentage of list price), c) manufacturing cost, d) operating expense (as a percentage), e) net profit after tax (as a percentage) may be calculated if the other four are known.

Since the tax range varies from company to company, provision is made for inputting your applicable tax rate. The example problem uses a tax rate of 48%.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f [P/R]</td>
<td>00-</td>
</tr>
<tr>
<td>f [CLEAR PRGM]</td>
<td>01-  45  5</td>
</tr>
<tr>
<td>RCL 5</td>
<td>02-  45  6</td>
</tr>
<tr>
<td>RCL 6</td>
<td>03-   10</td>
</tr>
<tr>
<td>+</td>
<td>04-   45  4</td>
</tr>
<tr>
<td>RCL 4</td>
<td>05-   40</td>
</tr>
<tr>
<td>CHS</td>
<td>06-   16</td>
</tr>
<tr>
<td>RCL 0</td>
<td>07-   45  0</td>
</tr>
<tr>
<td>+</td>
<td>08-   40</td>
</tr>
<tr>
<td>RCL 0</td>
<td>09-   45  0</td>
</tr>
<tr>
<td>+</td>
<td>10-   10</td>
</tr>
<tr>
<td>g [GTO]</td>
<td>11-43, 33 00</td>
</tr>
<tr>
<td>RCL 3</td>
<td>12-   45  3</td>
</tr>
<tr>
<td>RCL 1</td>
<td>13-   45  1</td>
</tr>
<tr>
<td>RCL 2</td>
<td>14-   45  2</td>
</tr>
<tr>
<td>RCL 0</td>
<td>15-   45  0</td>
</tr>
<tr>
<td>+</td>
<td>16-   10</td>
</tr>
<tr>
<td>CHS</td>
<td>17-   16</td>
</tr>
<tr>
<td>÷</td>
<td>18-   1</td>
</tr>
<tr>
<td>+</td>
<td>19-   40</td>
</tr>
<tr>
<td>×</td>
<td>20-   20</td>
</tr>
<tr>
<td>R/S</td>
<td>21-   31</td>
</tr>
<tr>
<td>÷</td>
<td>22-   10</td>
</tr>
<tr>
<td>Operation</td>
<td>Code 1</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
</tr>
<tr>
<td>CHS</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>+</td>
<td>25</td>
</tr>
<tr>
<td>RCL 0</td>
<td>26</td>
</tr>
<tr>
<td>x</td>
<td>27</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>28</td>
</tr>
<tr>
<td>+</td>
<td>29</td>
</tr>
<tr>
<td>CHS</td>
<td>30</td>
</tr>
<tr>
<td>RCL 1</td>
<td>31</td>
</tr>
<tr>
<td>+</td>
<td>32</td>
</tr>
<tr>
<td>RCL 1</td>
<td>33</td>
</tr>
<tr>
<td>+</td>
<td>34</td>
</tr>
<tr>
<td>RCL 0</td>
<td>35</td>
</tr>
<tr>
<td>x</td>
<td>36</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>37</td>
</tr>
<tr>
<td>RCL 5</td>
<td>38</td>
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<tr>
<td>RCL 6</td>
<td>39</td>
</tr>
<tr>
<td>+</td>
<td>40</td>
</tr>
<tr>
<td>-</td>
<td>41</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>42</td>
</tr>
<tr>
<td>RCL 4</td>
<td>43</td>
</tr>
<tr>
<td>-</td>
<td>44</td>
</tr>
<tr>
<td>RCL 6</td>
<td>45</td>
</tr>
<tr>
<td>x</td>
<td>46</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>47</td>
</tr>
<tr>
<td>f P/R</td>
<td></td>
</tr>
</tbody>
</table>

**REGISTERS**

n: Unused  
i: Unused  
PV: Unused  
PMT: Unused
1. Key in the program and press \texttt{REG}, then key in 100 and press \texttt{STO} 0.

2. Key in 1 and press \texttt{ENTER}, then key in your appropriate tax rate as a decimal and press \texttt{STO} 6.

3. a. Key in the list price in dollars (if known) and press \texttt{STO} 1.
   b. Key in the discount in percent (if known) and press \texttt{STO} 2.
   c. Key in the manufacturing cost in dollars (if known) and press \texttt{STO} 3.
   d. Key in the operating expense in percent (if known) and press \texttt{STO} 4.
   e. Key in the net profit after tax in percent (if known) and press \texttt{STO} 5.

4. To calculate list price:
   a. Do steps 2 and 3b, c, d, e above.
   b. Press \texttt{RCL 3 R/S ÷ 1 g GTO 14 R/S ÷ g GTO 00}.

5. To calculate discount:
   a. Do steps 2 and 3a, c, d, e above.
   b. Press \texttt{RCL 3 R/S g GTO 29 R/S}.

6. To calculate manufacturing cost:
   a. Do steps 2 and 3a, b, d, e, above.
   b. Press \texttt{g GTO 13 R/S g GTO 01 R/S x}.

7. To calculate operating expense:
   a. Do steps 2 and 3a, b, c, e, above.
   b. Press \texttt{g GTO 12 R/S R/S g GTO 38 R/S}.

8. To calculate net profit after tax:
   a. Do steps 2 and 3a, b, c, d, above.

FV: Unused  
\( R_0: 100 \)

\( R_1: \) list price  
\( R_2: \% \) discount

\( R_3: \) mfg. cost  
\( R_4: \% \) op. exp.

\( R_5: \% \) net profit  
\( R_6: 1-\% \) tax

\( R_7-R_3: \) Unused
b. Press $9$ GTO $12$ R/S R/S $9$ GTO $43$ R/S.

**Example:** What is the net return on an item that is sold for $11.98$, discounted through distribution an average of 35% and has a manufacturing cost of $2.50$? The standard company operating expense is 32% of net shipping (sales) price and tax rate is 48%.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f CLEAR</td>
<td>REG</td>
</tr>
<tr>
<td>100 STO 0</td>
<td>100.00</td>
</tr>
<tr>
<td>1 ENTER 48</td>
<td>- STO 6</td>
</tr>
<tr>
<td>11.98 STO 1</td>
<td>11.98</td>
</tr>
<tr>
<td>35 STO 2</td>
<td>35.00</td>
</tr>
<tr>
<td>2.50 STO 3</td>
<td>2.50</td>
</tr>
<tr>
<td>32 STO 4</td>
<td>32.00</td>
</tr>
<tr>
<td>$9$ GTO 12</td>
<td>R/S R/S</td>
</tr>
<tr>
<td>67.90</td>
<td></td>
</tr>
<tr>
<td>$9$ GTO 43</td>
<td>R/S</td>
</tr>
<tr>
<td>18.67</td>
<td></td>
</tr>
</tbody>
</table>

If manufacturing expenses increase to $3.25, what is the effect on net profit?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.25 STO 3</td>
<td>3.25</td>
</tr>
<tr>
<td>$9$ GTO 12</td>
<td>R/S R/S</td>
</tr>
<tr>
<td>58.26</td>
<td></td>
</tr>
<tr>
<td>$9$ GTO 43</td>
<td>R/S</td>
</tr>
<tr>
<td>13.66 Net profit reduced to 13.66%</td>
<td></td>
</tr>
</tbody>
</table>

If the manufacturing cost is maintained at $3.25, how high could the overhead (operating expense) be before the product begins to lose money?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 STO 5</td>
<td>0.00</td>
</tr>
<tr>
<td>$9$ GTO 12</td>
<td>R/S R/S</td>
</tr>
<tr>
<td>58.26</td>
<td></td>
</tr>
<tr>
<td>$9$ GTO 38</td>
<td>R/S</td>
</tr>
<tr>
<td>58.26 Maximum operating expense (%)</td>
<td></td>
</tr>
</tbody>
</table>

At 32% operating expense and $3.25 manufacturing cost, what should the list price be to generate 20% net profit?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 STO 5</td>
<td>20.00</td>
</tr>
<tr>
<td>RCL 3 R/S 1</td>
<td>11.00</td>
</tr>
<tr>
<td>$1$ $9$ GTO 14 R/S 1</td>
<td></td>
</tr>
<tr>
<td>16.93 List price ($)</td>
<td></td>
</tr>
</tbody>
</table>
What reduction in manufacturing cost would achieve the same result without necessitating an increase in list price above $11.98?

<table>
<thead>
<tr>
<th>g</th>
<th>GTO</th>
<th>R/S</th>
<th>7.79</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>GTO</td>
<td>01</td>
<td>x    2.30</td>
</tr>
</tbody>
</table>
Securities

After-Tax Yield

The following HP-12C program calculates the after-tax yield to maturity of a bond held for more than one year. The calculations assume an actual/actual day basis. For after-tax computations, interest or coupon payments are considered income, while the difference between the bond or note face value and its purchase price is considered capital gains.

<table>
<thead>
<tr>
<th>KEYS</th>
<th>DISPAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td>00-</td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>01- 42 34</td>
</tr>
<tr>
<td>f CLEAR FIN</td>
<td>02- 44 7</td>
</tr>
<tr>
<td>STO 7</td>
<td>03- 33</td>
</tr>
<tr>
<td>R↓</td>
<td>04- 44 6</td>
</tr>
<tr>
<td>STO 6</td>
<td>05- 45 2</td>
</tr>
<tr>
<td>RCL 2</td>
<td>06- 45 1</td>
</tr>
<tr>
<td>RCL 4</td>
<td>07- 30</td>
</tr>
<tr>
<td>%</td>
<td>08- 45 4</td>
</tr>
<tr>
<td>RCL 2</td>
<td>09- 25</td>
</tr>
<tr>
<td>÷</td>
<td>10- 45 2</td>
</tr>
<tr>
<td>2</td>
<td>11- 34</td>
</tr>
<tr>
<td>√</td>
<td>12- 30</td>
</tr>
<tr>
<td>EEX</td>
<td>13- 26</td>
</tr>
<tr>
<td>2</td>
<td>14- 2</td>
</tr>
<tr>
<td>÷</td>
<td>15- 10</td>
</tr>
<tr>
<td>STO 0</td>
<td>16- 44 0</td>
</tr>
<tr>
<td>RCL 3</td>
<td>17- 45 3</td>
</tr>
<tr>
<td>RCL 5</td>
<td>18- 45 5</td>
</tr>
</tbody>
</table>
1. Key in the program.

2. Key in the purchase price and press \( \text{STO} \) 1.

3. Key in the sales price and press \( \text{STO} \) 2.

4. Key in the annual coupon rate (as a percentage) and press \( \text{STO} \) 3.

5. Key in capital gains tax rate (as a percentage) and press \( \text{STO} \) 4.

6. Key in the income tax rate (as a percentage) and press \( \text{STO} \) 5.

7. Press \( \text{g} \) \( \text{M.DY} \).
8. Key in the purchase date (MM.DDYYYY) and press ENTER.
9. Key in the assumed sell date (MM.DDYYYY) and press R/S to find the after-tax yield (as a percentage).
10. For the same bond but different date return to step 8.
11. For a new case return to step 2.

Example: You can buy a 7% bond on October 1, 1981 for $70 and expect to sell it in 5 years for $90. What is your net (after-tax) yield over the 5-year period if interim coupon payments are considered as income, and your tax bracket is 50%?

(One-half of the long term capital gain is taxable at 50%, so the tax on capital gains alone is 25%)

### Keystrokes and Display

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 STO 1</td>
<td>10.00</td>
</tr>
<tr>
<td>90 STO 2</td>
<td>90.00</td>
</tr>
<tr>
<td>7 STO 3</td>
<td>7.00</td>
</tr>
<tr>
<td>25 STO 4</td>
<td>25.00</td>
</tr>
<tr>
<td>50 STO 5</td>
<td>50.00</td>
</tr>
<tr>
<td>9 M.DY</td>
<td>10.01</td>
</tr>
<tr>
<td>10.011981 R/S</td>
<td>8.53</td>
</tr>
</tbody>
</table>

**Discounted Notes**

A note is a written agreement to pay a sum of money plus interest at a certain rate. Notes to not have periodic coupons, since all interest is paid at maturity.

A discounted note is a note that is purchase below its face value. The following HP 12C program finds the price and/or yield* (*The yield is a reflection of the return on an investment) of a discounted note.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td></td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>00-</td>
</tr>
<tr>
<td>RCL 1</td>
<td>01- 45 1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-------</td>
</tr>
<tr>
<td>RCL₃</td>
<td>02-</td>
</tr>
<tr>
<td>g [ΔYS]</td>
<td>03-</td>
</tr>
<tr>
<td>RCL₅</td>
<td>04-</td>
</tr>
<tr>
<td>+</td>
<td>05-</td>
</tr>
<tr>
<td>RCL₄</td>
<td>06-</td>
</tr>
<tr>
<td>%</td>
<td>07-</td>
</tr>
<tr>
<td>1</td>
<td>08-</td>
</tr>
<tr>
<td>x∔y</td>
<td>09-</td>
</tr>
<tr>
<td>-</td>
<td>10-</td>
</tr>
<tr>
<td>RCL₃</td>
<td>11-</td>
</tr>
<tr>
<td>x</td>
<td>12-</td>
</tr>
<tr>
<td>STO₅</td>
<td>13-</td>
</tr>
<tr>
<td>R/S</td>
<td>14-</td>
</tr>
<tr>
<td>RCL₁</td>
<td>15-</td>
</tr>
<tr>
<td>RCL₂</td>
<td>16-</td>
</tr>
<tr>
<td>g [ΔYS]</td>
<td>17-</td>
</tr>
<tr>
<td>RCL₃</td>
<td>18-</td>
</tr>
<tr>
<td>x∔y</td>
<td>19-</td>
</tr>
<tr>
<td>+</td>
<td>20-</td>
</tr>
<tr>
<td>RCL₄</td>
<td>21-</td>
</tr>
<tr>
<td>RCL₅</td>
<td>22-</td>
</tr>
<tr>
<td>+</td>
<td>23-</td>
</tr>
<tr>
<td>1</td>
<td>24-</td>
</tr>
<tr>
<td>-</td>
<td>25-</td>
</tr>
<tr>
<td>x</td>
<td>26-</td>
</tr>
<tr>
<td>EEEX</td>
<td>27-</td>
</tr>
<tr>
<td>2</td>
<td>28-</td>
</tr>
<tr>
<td>x</td>
<td>29-</td>
</tr>
<tr>
<td>g [GTO]₀₀</td>
<td>30-</td>
</tr>
</tbody>
</table>
1. Key in the program.

2. Press [g] M.D.Y.


4. Key in the maturity date (MM.DDYYYY) and press [STO] 2.

5. Key in the number of days in a year (360 or 365) and press [STO] 3.


7. To calculate the purchase price:
   b. Press [R/S] to calculate the purchase price.
   c. Press [R/S] to calculate the yield.
   d. For a new case, go to step 3.

8. To calculate the yield when the price is known:
   b. Press [g] GTO 15 [R/S] to calculate the yield.
   c. For a new case, go to step 3.

**Example 1:** Calculate the price and yield on this U.S. Treasury Bill: settlement date October 8, 1980; maturity date March 21, 1981; discount rate 7.80%. Compute on a 360 day basis.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>[g] M.D.Y</td>
<td>10.08</td>
</tr>
<tr>
<td>10.081920 [STO] 1</td>
<td>Settlement date.</td>
</tr>
</tbody>
</table>
### Example 2: Determine the yield of this security; settlement date June 25, 1980; maturity date September 10, 1980; price $99.45; redemption value $101.33. Assume 360 day basis.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.10 1980</td>
<td>9.10 Maturity date.</td>
</tr>
<tr>
<td>360</td>
<td>360.00 360 day basis.</td>
</tr>
<tr>
<td>101.33</td>
<td>101.33 Redemption value per $100.</td>
</tr>
<tr>
<td>99.45</td>
<td>99.45 Price.</td>
</tr>
<tr>
<td>8.84</td>
<td>8.84 Yield.</td>
</tr>
</tbody>
</table>

- **Keystrokes:**
  - 3.21 1981 STO 2 3.21 Maturity date.
  - 360 STO 3 360.00 360 day basis.
  - 100 STO 4 100.00 Redemption value per $100.
  - 7.8 STO 5 7.80 Discount rate.
  - R/S 96.45 Price.
  - R/S 8.09 Yield.
Forecasting

Simple Moving Average

Moving averages are often useful in recording of forecasting sales figures, expenses or manufacturing volume. There are many different types of moving average calculations. An often used, straightforward method of calculation is presented here.

In a moving average a specified number of data points are averaged. When there is a new piece of input data, the oldest piece of data is discarded to make room for the latest input. This replacement scheme makes the moving average a valuable tool in following trends. The fewer the number of data points, the more trend sensitive the average becomes. With a large number of data points, the average behaves more like a regular average, responding slowly to new input data.

A simple moving average may be calculated with your HP 12C as follows.

1. Press **f CLEAR REG**.
2. Key in the first \( m \) data points (where \( m \) is the number of data points in the average) and press \( \sum+ \) after each entry.
3. Press \( \sum\bar{x} \) to obtain the first average.
4. Key in the oldest (first value) entered in step 2 and press \( \sum- \).
5. Key in the newest data point \( (m + 1) \) and press \( \sum+ \).
6. Press \( \sum\bar{x} \) to obtain the next value of the moving average.
7. Repeat steps 4 through 5 for the remaining data.

**Example:** An electronics sales firm wished to calculate a 3-month moving average for the dollar volume of components sold each month. Sales for the first six months of this year were:

<table>
<thead>
<tr>
<th>Month</th>
<th>Dollar Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>$211,570</td>
</tr>
<tr>
<td>February</td>
<td>$112,550</td>
</tr>
<tr>
<td>March</td>
<td>$190,060</td>
</tr>
<tr>
<td>April</td>
<td>$131,760</td>
</tr>
<tr>
<td>May</td>
<td>$300,500</td>
</tr>
<tr>
<td>June</td>
<td>$271,120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For repeated calculations the following HP 12C program can be used for up to a 12 element moving average:

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEAR REG</td>
<td>0.00</td>
</tr>
<tr>
<td>211570 Σ+</td>
<td>1.00</td>
</tr>
<tr>
<td>112550 Σ+</td>
<td>2.00</td>
</tr>
<tr>
<td>190060 Σ+</td>
<td>3.00</td>
</tr>
<tr>
<td>g x</td>
<td>171,393.33 3-month average for March.</td>
</tr>
<tr>
<td>211570 g Σ-</td>
<td>2.00</td>
</tr>
<tr>
<td>131760 Σ+</td>
<td>3.00</td>
</tr>
<tr>
<td>g x</td>
<td>144,790.00 3-month average for April.</td>
</tr>
<tr>
<td>112550 g Σ-</td>
<td>2.00</td>
</tr>
<tr>
<td>300500 Σ+</td>
<td>3.00</td>
</tr>
<tr>
<td>g x</td>
<td>207,440.00 3-month average for May.</td>
</tr>
<tr>
<td>190060 g Σ-</td>
<td>2.00</td>
</tr>
<tr>
<td>271120 Σ+</td>
<td>3.00</td>
</tr>
<tr>
<td>g x</td>
<td>234,460.00 3-month average for June.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>+</td>
<td>4</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>5</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>4</td>
</tr>
<tr>
<td>+</td>
<td>5</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>5</td>
</tr>
<tr>
<td>+</td>
<td>6</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>7</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>6</td>
</tr>
<tr>
<td>+</td>
<td>7</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>8</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>7</td>
</tr>
<tr>
<td>+</td>
<td>8</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>9</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>9</td>
</tr>
<tr>
<td>+</td>
<td>9</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>0</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>9</td>
</tr>
<tr>
<td>+</td>
<td>10</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>1</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>0</td>
</tr>
<tr>
<td>+</td>
<td>11</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>2</td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td>1</td>
</tr>
<tr>
<td>+</td>
<td>12</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td>0</td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>
At step 38, \( m \) = number of elements in the moving average, i.e., for a 5-element moving average line 38 would be \( n \) \( \text{STO} \) 5 and for a 12-element average line 38 would be \( n \) \( \text{STO} \) 2.

This program can be used for a moving average of 2 to 12 elements. It may be shortened considerably for moving averages with less than 12 elements. To do this, key in the program, as shown, form line 01 until you reach a superscripted with the number of elements you desire. Key in this line, then skip the rest of the program down to line 35. Then key in lines 35 through 39, being sure to specify the register number at line 38, \( n \) \( \text{STO} \) \( m \), corresponding to the number of elements you are using. (For instance, for a 5 element moving average, key in lines 01 through 13 then go to line 35 in the listing and key in the balance of the program. Obviously the program listing line 38, \( n \) \( \text{STO} \) \( m \), becomes the displayed line 17, \( n \) \( \text{STO} \) 5).

To run the program:

1. Key in the program.
2. Press \( f \text{ CLEAR} \) \( \text{REG} \). Key in the number of elements, \( m \), and press the number of elements, \( m \), and press 0.
3. Key in the second data point and press \( n \) \( \text{STO} \) 1.
4. Key in the second data point and press \( n \) \( \text{STO} \) 2.
5. Continue as above, keying in and storing each data point in its appropriate register until \( m \) data points have been stored.

6. Press \( \texttt{g GTO 00 R/S} \) to calculate the first moving average.

7. Key in the next data point and press \( \texttt{R/S} \) to calculate the next moving average.

8. Repeat step 7 for each new data point.

**Example 2:** Calculate the 3-element moving average for the data given in example 1. Your modified program listing will look like this:

<table>
<thead>
<tr>
<th>KEYS</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) ( \text{P/R} )</td>
<td>00-</td>
</tr>
<tr>
<td>( f ) ( \text{CLEAR PRGM} )</td>
<td>01- 45 1</td>
</tr>
<tr>
<td>( \text{RCL} ) 1</td>
<td>02- 45 2</td>
</tr>
<tr>
<td>( \text{RCL} ) 2</td>
<td>03- 44 1</td>
</tr>
<tr>
<td>( + )</td>
<td>04- 40</td>
</tr>
<tr>
<td>( \text{RCL} ) 3</td>
<td>05- 45 3</td>
</tr>
<tr>
<td>( \text{STO} ) 1</td>
<td>06- 44 2</td>
</tr>
<tr>
<td>3</td>
<td>07- 40</td>
</tr>
<tr>
<td>( \text{RCL} ) 0</td>
<td>08- 45 0</td>
</tr>
<tr>
<td>( + )</td>
<td>09- 10</td>
</tr>
<tr>
<td>( \text{R/S} )</td>
<td>10- 31</td>
</tr>
<tr>
<td>( \text{STO} ) 3</td>
<td>11- 44 3</td>
</tr>
<tr>
<td>( \texttt{g GTO 01} )</td>
<td>12-43, 33 01</td>
</tr>
<tr>
<td>( f ) ( \text{P/R} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) ( \text{CLEAR REG} )</td>
<td>0.00</td>
</tr>
<tr>
<td>3 ( \text{STO} ) 0</td>
<td>3.00</td>
</tr>
<tr>
<td>211570 ( \text{STO} ) 1</td>
<td>211,570.00</td>
</tr>
<tr>
<td>112550 ( \text{STO} ) 2</td>
<td>112,550.00</td>
</tr>
<tr>
<td>190060 ( \text{STO} ) 3</td>
<td>190,060.00</td>
</tr>
</tbody>
</table>
Seasonal Variation Factors Based on Centered Moving Averages.

Seasonal variation factors are useful concepts in many types of forecasting. There are several methods of developing seasonal moving averages, one of which is to calculate them as a ratio of the periodic value to a centered moving average for the same period.

For instance, to determine the sales for the 3rd quarter of a given year a centered moving average for that quarter would be calculated from sales figures from the 1st, 2nd, 3rd and 4th quarters of the year and the 1st quarter of the following year. The seasonal variation factor for that 3rd quarter would then be the ratio of the actual sales in the 3rd quarter to the centered moving average for that quarter.

While quarterly seasonal variations are commonly used, the HP 12C can also be programmed to calculate monthly seasonal variations using a centered 12 month moving averages. Programs for both of these calculations are represented here:

An HP 12C program to calculate the quarterly seasonal variations based on a centered 4-point moving average is:

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>P R</td>
<td>171,393.33 3-month average for March.</td>
</tr>
<tr>
<td>131760 R/S</td>
<td>144,790.00 3-month average for April.</td>
</tr>
<tr>
<td>300500 R/S</td>
<td>207,440.00 3-month average for May.</td>
</tr>
<tr>
<td>271120 R/S</td>
<td>234,460.00 3-month average for June.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P R</td>
<td>00 -</td>
</tr>
<tr>
<td>f CLEAR PRG</td>
<td>01 - 45 1</td>
</tr>
<tr>
<td>RCL 1 2</td>
<td>02 - 2</td>
</tr>
<tr>
<td>+</td>
<td>03 - 10</td>
</tr>
<tr>
<td>RCL 2</td>
<td>04 - 45 2</td>
</tr>
<tr>
<td>STO 1</td>
<td>05 - 44 1</td>
</tr>
<tr>
<td>+</td>
<td>06 - 40</td>
</tr>
<tr>
<td>RCL 3</td>
<td>07 - 45 3</td>
</tr>
</tbody>
</table>
1. Key in the program.

2. Press \texttt{f CLEAR REG}.

3. Key in the quarterly sales figures starting with the first quarter:
   a. Key in 1st quarter sales and press \texttt{STO 1}.

\begin{tabular}{|c|c|}
\hline
\texttt{STO 2} & 08 - 44 2 \\
\hline
\texttt{RCL 4} & 10 - 45 4 \\
\hline
\texttt{STO 3} & 11 - 44 3 \\
\hline
\texttt{RCL 5} & 13 - 45 5 \\
\hline
\texttt{STO 4} & 14 - 44 4 \\
\hline
\texttt{STO 5} & 15 - 44 5 \\
\hline
\texttt{GTO 01} & 16 - 23 \\
\hline
\texttt{P/R} & 17 - 40 \\
\hline
\end{tabular}

\textbf{REGISTERS}

\begin{itemize}
  \item n: Unused
  \item i: Unused
  \item PV: Unused
  \item PMT: Unused
  \item FV: Unused
  \item R0: n
  \item R1: X1
  \item R2: X2
  \item R3: X3
  \item R4: X4
  \item R5: X5
  \item R6: Unused
\end{itemize}
b. Key in 2nd quarter sales and press $\text{STO} \underline{2}$.

c. Key in 3rd quarter sales and press $\text{STO} \underline{3}$.

d. Key in 4th quarter sales and press $\text{STO} \underline{4}$.

e. Key in the 1st quarter sales for the next year and press $\text{STO} \underline{5}$.

4. Press $9 \text{GTO} \underline{00} \text{ R/S}$ to calculate the centered moving average for the 3rd quarter of the first year.

5. Press $\text{R/S}$ to calculate the seasonal variation for this quarter.

6. Key in the next quarter's sales and press $\text{R/S}$ to calculate the moving average for the next quarter.

7. Press $\text{R/S}$ to calculate the seasonal variation.

8. Repeat steps 6 and 7 for the balance of the data.

**Example:** Econo-Wise Home Appliance Company had quarterly sales for the years 1978 thru 1980 as follows:

<table>
<thead>
<tr>
<th>Sales (IN $K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarterly</td>
</tr>
<tr>
<td>1978</td>
</tr>
<tr>
<td>1979</td>
</tr>
<tr>
<td>1980</td>
</tr>
</tbody>
</table>

Find the centered 4-quarter moving average and seasonal variation factor for each quarter.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{f \ CLEAR \ REG}$</td>
<td>0.00</td>
</tr>
<tr>
<td>397 $\text{STO} \underline{1}$</td>
<td>397.00</td>
</tr>
<tr>
<td>376 $\text{STO} \underline{2}$</td>
<td>376.00</td>
</tr>
<tr>
<td>460 $\text{STO} \underline{3}$</td>
<td>460.00</td>
</tr>
<tr>
<td>501 $\text{STO} \underline{4}$</td>
<td>501.00</td>
</tr>
<tr>
<td>455 $\text{STO} \underline{5}$</td>
<td>455.00</td>
</tr>
<tr>
<td>$9 \text{GTO} \underline{00} \text{ R/S}$</td>
<td>440.75</td>
</tr>
<tr>
<td>$\text{R/S}$</td>
<td>104.37</td>
</tr>
</tbody>
</table>
Now average each quarter's seasonal variation for the two years?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f CLEAR Σ</td>
<td>0.00</td>
</tr>
<tr>
<td>98.86 Σ+</td>
<td>1.00</td>
</tr>
<tr>
<td>99.95 Σ+</td>
<td>2.00</td>
</tr>
</tbody>
</table>
| g X            | 99.41 1st quarter average seasonal variation, %.
| f CLEAR Σ     | 0.00                                         |
| 81.87 Σ+       | 1.00                                         |
| 83.24 Σ+       | 2.00                                         |
| g X            | 82.56 2nd quarter average seasonal variation, %.
| f CLEAR Σ     | 0.00                                         |
| 104.37 Σ+      | 1.00                                         |
| 107.94 Σ+      | 2.00                                         |
| g X            | 106.16 3rd quarter average seasonal variation, %.

79
An HP-12C program to calculate a centered 12-month moving average and seasonal variation factor is as follows:

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>f</code> P/R</td>
<td>00-</td>
</tr>
<tr>
<td><code>f</code> CLEAR PRGM</td>
<td>01- 45 1</td>
</tr>
<tr>
<td>RCL 1</td>
<td>02- 2</td>
</tr>
<tr>
<td>+</td>
<td>03- 10</td>
</tr>
<tr>
<td>RCL 2</td>
<td>04- 45 2</td>
</tr>
<tr>
<td>STO 1</td>
<td>05- 44 1</td>
</tr>
<tr>
<td>+</td>
<td>06- 40</td>
</tr>
<tr>
<td>RCL 3</td>
<td>07- 45 3</td>
</tr>
<tr>
<td>STO 2</td>
<td>08- 44 2</td>
</tr>
<tr>
<td>+</td>
<td>09- 40</td>
</tr>
<tr>
<td>RCL 4</td>
<td>10- 45 4</td>
</tr>
<tr>
<td>STO 3</td>
<td>11- 44 3</td>
</tr>
<tr>
<td>+</td>
<td>12- 40</td>
</tr>
<tr>
<td>RCL 5</td>
<td>13- 45 5</td>
</tr>
<tr>
<td>STO 4</td>
<td>14- 44 4</td>
</tr>
<tr>
<td>+</td>
<td>15- 40</td>
</tr>
<tr>
<td>RCL 6</td>
<td>16- 45 6</td>
</tr>
<tr>
<td>STO 5</td>
<td>17- 44 5</td>
</tr>
<tr>
<td>+</td>
<td>18- 40</td>
</tr>
<tr>
<td>RCL 7</td>
<td>19- 45 7</td>
</tr>
</tbody>
</table>

4th quarter average seasonal variation, %.
<table>
<thead>
<tr>
<th>Command</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>STO 6</td>
<td>20- 44 6</td>
</tr>
<tr>
<td>+</td>
<td>21- 40</td>
</tr>
<tr>
<td>RCL 8</td>
<td>22- 45 8</td>
</tr>
<tr>
<td>STO 7</td>
<td>23- 44 7</td>
</tr>
<tr>
<td>+</td>
<td>24- 40</td>
</tr>
<tr>
<td>RCL 9</td>
<td>25- 45 9</td>
</tr>
<tr>
<td>STO 8</td>
<td>26- 44 8</td>
</tr>
<tr>
<td>+</td>
<td>27- 40</td>
</tr>
<tr>
<td>RCL 0</td>
<td>28-45 48 0</td>
</tr>
<tr>
<td>STO 9</td>
<td>29- 44 9</td>
</tr>
<tr>
<td>+</td>
<td>30- 40</td>
</tr>
<tr>
<td>RCL 1</td>
<td>31-45 48 1</td>
</tr>
<tr>
<td>STO 0</td>
<td>32-44 48 0</td>
</tr>
<tr>
<td>+</td>
<td>33- 40</td>
</tr>
<tr>
<td>RCL 2</td>
<td>34-45 48 2</td>
</tr>
<tr>
<td>STO 1</td>
<td>35-44 48 1</td>
</tr>
<tr>
<td>+</td>
<td>36- 40</td>
</tr>
<tr>
<td>RCL 3</td>
<td>37-45 48 3</td>
</tr>
<tr>
<td>STO 2</td>
<td>38-44 48 2</td>
</tr>
<tr>
<td>2</td>
<td>39- 2</td>
</tr>
<tr>
<td>+</td>
<td>40- 10</td>
</tr>
<tr>
<td>+</td>
<td>41- 40</td>
</tr>
<tr>
<td>RCL 0</td>
<td>42- 45 0</td>
</tr>
<tr>
<td>+</td>
<td>43- 10</td>
</tr>
<tr>
<td>R/S</td>
<td>44- 31</td>
</tr>
<tr>
<td>RCL 6</td>
<td>45- 45 6</td>
</tr>
<tr>
<td>%T</td>
<td>46- 23</td>
</tr>
<tr>
<td>R/S</td>
<td>47- 31</td>
</tr>
</tbody>
</table>
1. Key in the program.
2. Press \[ \text{f} \quad \text{CLEAR} \quad \text{REG} \]
3. Key in 12 and press 0.
4. Key in the values for the first 13 months, storing them one at a time in registers 1 through .3, i.e.
   - Key in the 1st month and press \[ \text{STO} \quad 1 \]
   - Key in the 2nd month and press \[ \text{STO} \quad 2 \]
   - Key in the 10th month and press \[ \text{STO} \quad 0 \]
   - Key in the 13th month and press \[ \text{STO} \quad 3 \]
5. Press \[ \text{g} \quad \text{GTO} \quad 00 \quad \text{R/S} \] to calculate the centered moving average for the 7th month.
6. Press \[ \text{R/S} \] to calculate the seasonal variation for that month.
7. Key in the value for the next month (14th) and press \[ \text{R/S} \] to calculate the moving average for the next month (8th).
8. Repeat steps 6 and 7 for the balance of the data.

These programs may be customized by the user for different types of centered moving averages. Inspection of the programs will show how they can be modified.

Gompertz Curve Trend Analysis
A useful curve for evaluating sales trends, etc., is the Gompertz curve. This is a "growth" curve having a general "S" shape and may be used to describe series of data where the early rate of growth is small, then accelerates for a period of time and then slows again as the time grows long. The sales curve for many products follow this trend during the introductory, growth and maturity phases.

The data points to be fit to a Gompertz curve should be equally spaced along the x (or time) axis and all the data points must be positive. The points are divided serially into 3 groups for data entry.

The following HP 12C program processes the data, fits it to a Gompertz curve and calculates estimated values for future data points. The 3 constants which characterize the curve are available to the user if desired.

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td>00-</td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>01- 43 23</td>
</tr>
<tr>
<td>g LN</td>
<td>02-44 40 3</td>
</tr>
<tr>
<td>R↓</td>
<td>03- 33</td>
</tr>
<tr>
<td>g LN</td>
<td>04- 43 23</td>
</tr>
<tr>
<td>STO 3</td>
<td>05-44 40 2</td>
</tr>
<tr>
<td>R↓</td>
<td>06- 33</td>
</tr>
<tr>
<td>g R↓</td>
<td>07- 43 23</td>
</tr>
<tr>
<td>STO 1</td>
<td>08-44 40 1</td>
</tr>
<tr>
<td>R↓</td>
<td>09- 1</td>
</tr>
<tr>
<td>STO 4</td>
<td>10-44 40 4</td>
</tr>
<tr>
<td>RCL 4</td>
<td>11- 45 4</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>12-43, 33 00</td>
</tr>
<tr>
<td>RCL 3</td>
<td>13- 45 3</td>
</tr>
<tr>
<td>RCL 2</td>
<td>14- 45 2</td>
</tr>
<tr>
<td>-</td>
<td>15- 30</td>
</tr>
<tr>
<td>RCL 2</td>
<td>16- 45 2</td>
</tr>
<tr>
<td>RCL 1</td>
<td>17- 45 1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>18-</td>
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</tr>
<tr>
<td>19-</td>
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</tr>
<tr>
<td>25-</td>
<td>45</td>
</tr>
<tr>
<td>26-</td>
<td>20</td>
</tr>
<tr>
<td>27-</td>
<td>45</td>
</tr>
<tr>
<td>28-</td>
<td>36</td>
</tr>
<tr>
<td>29-</td>
<td>20</td>
</tr>
<tr>
<td>30-</td>
<td>30</td>
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<td>31-</td>
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<td>32-</td>
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</tr>
<tr>
<td>35-</td>
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<td>36-</td>
<td>20</td>
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<td>37-</td>
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<td>38-</td>
<td>10</td>
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<tr>
<td>39-</td>
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</tr>
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<td>40-</td>
<td>10</td>
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<td>45-</td>
<td>30</td>
</tr>
<tr>
<td>Key</td>
<td>Function</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
</tr>
<tr>
<td>RCL 6</td>
<td>46 - 45 6</td>
</tr>
<tr>
<td>RCL 4</td>
<td>47 - 45 4</td>
</tr>
<tr>
<td>RCL 1</td>
<td>48 - 21</td>
</tr>
<tr>
<td>STO</td>
<td>49 - 1</td>
</tr>
<tr>
<td>t</td>
<td>50 - 30</td>
</tr>
<tr>
<td>x</td>
<td>51 - 36</td>
</tr>
<tr>
<td>+</td>
<td>52 - 20</td>
</tr>
<tr>
<td>-</td>
<td>53 - 10</td>
</tr>
<tr>
<td>RCL 6</td>
<td>54 - 45 6</td>
</tr>
<tr>
<td>RCL 2</td>
<td>55 - 10</td>
</tr>
<tr>
<td>RCL 1</td>
<td>56 - 45 2</td>
</tr>
<tr>
<td>R/S</td>
<td>57 - 45 1</td>
</tr>
<tr>
<td>x</td>
<td>58 - 30</td>
</tr>
<tr>
<td>e^x</td>
<td>59 - 20</td>
</tr>
<tr>
<td>g</td>
<td>60 - 43 22</td>
</tr>
<tr>
<td>STO</td>
<td>61 - 44 5</td>
</tr>
<tr>
<td>R/S</td>
<td>62 - 31</td>
</tr>
<tr>
<td>RCL</td>
<td>63 - 45 6</td>
</tr>
<tr>
<td>RCL 5</td>
<td>64 - 34</td>
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<tr>
<td>y^x</td>
<td>65 - 21</td>
</tr>
<tr>
<td>RCL 7</td>
<td>66 - 45 5</td>
</tr>
<tr>
<td>RCL 5</td>
<td>67 - 34</td>
</tr>
<tr>
<td>RCL 1</td>
<td>68 - 21</td>
</tr>
<tr>
<td>7</td>
<td>69 - 45 7</td>
</tr>
<tr>
<td>x</td>
<td>70 - 20</td>
</tr>
<tr>
<td>g</td>
<td>71-43, 33 62</td>
</tr>
<tr>
<td>P/R</td>
<td></td>
</tr>
</tbody>
</table>

**REGISTERS**

n: Unused  
i: Unused
1. Key in the program and press \( \text{f} \) CLEAR \( \text{REG} \).
2. Divide the data points to be input into 3 equal consecutive groups. Label them Groups I, II and III for convenience.
3. Key in the first point of group I and press \( \text{ENTER} \).
4. Key in the first point of group II and press \( \text{ENTER} \).
5. Key in the first point of group III and press \( \text{R/S} \).
6. Repeat steps 3, 4, and 5 for the balance of the data in each group. After executing step 5 the display shows how many sets of data have been entered.
7. To fit the data to a Gompertz curve, press \( \text{8} \) GTO \( 12 \text{R/S} \). The resultant display is the curve constant "a". Constants "b" and "c" may be obtained by pressing \( \text{RCL} \) 6 and \( \text{RCL} \) 7 respectively.
8. To calculate a projected value, key in the number of the period and press \( \text{R/S} \).
9. Repeat step 8 for each period desired.

**Example:** The X-presso Company marked a revolutionary new coffee brewing machine in 1968. Sales grew at a steady pace for several years, then began to slow. The sales records for the first 9 years of the product's life were as follows.

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales($K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>41</td>
</tr>
<tr>
<td>3</td>
<td>49</td>
</tr>
<tr>
<td>4</td>
<td>151</td>
</tr>
<tr>
<td>5</td>
<td>188</td>
</tr>
<tr>
<td>6</td>
<td>260</td>
</tr>
<tr>
<td>7</td>
<td>282</td>
</tr>
<tr>
<td>8</td>
<td>322</td>
</tr>
<tr>
<td>9</td>
<td>340</td>
</tr>
</tbody>
</table>

What are the projected sales volumes for this product in its 10th and 12th year? What is the maximum yearly sales volume for this product if the
present trend continues? What annual sales rate would the curve have predicted for the 5th year of the product's life? (Arrange the data as follows:)

<table>
<thead>
<tr>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>151</td>
<td>282</td>
</tr>
<tr>
<td>41</td>
<td>188</td>
<td>322</td>
</tr>
<tr>
<td>49</td>
<td>260</td>
<td>340</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) CLEAR REG</td>
<td>0.00</td>
</tr>
<tr>
<td>18 ENTER</td>
<td>18.00</td>
</tr>
<tr>
<td>151 ENTER</td>
<td>151.00</td>
</tr>
<tr>
<td>282 R/S</td>
<td>1.00</td>
</tr>
<tr>
<td>41 ENTER</td>
<td>41.00</td>
</tr>
<tr>
<td>188 ENTER</td>
<td>188.00</td>
</tr>
<tr>
<td>322 R/S</td>
<td>2.00</td>
</tr>
<tr>
<td>49 ENTER</td>
<td>49.00</td>
</tr>
<tr>
<td>260 ENTER</td>
<td>260.00</td>
</tr>
<tr>
<td>340 R/S</td>
<td>3.00 Total number of entries.</td>
</tr>
<tr>
<td>( g ) GTO 13 R/S</td>
<td>0.004 ( a )</td>
</tr>
<tr>
<td>RCL 6</td>
<td>0.65 ( b )</td>
</tr>
<tr>
<td>RCL 7</td>
<td>373.92 ( c )</td>
</tr>
<tr>
<td>10 R/S</td>
<td>349.09 Sales in 10th year, (in $K).</td>
</tr>
<tr>
<td>12 R/S</td>
<td>363.36 Sales in 12th year, (in $K).</td>
</tr>
<tr>
<td>100 R/S</td>
<td>373.92 Maximum annual sales (after very long product life).</td>
</tr>
<tr>
<td>5 R/S</td>
<td>202.60 Sales in 5th year (actual sales were $188K).</td>
</tr>
</tbody>
</table>

**Forecasting with Exponential Smoothing**

A common method for analyzing trends in sales, inventory and securities is the moving average. Exponential smoothing is a version of the weighted moving average which is readily adaptable to programmable calculator forecasting.
Exponential smoothing is often used for short term sales and inventory forecasts. Typical forecast periods are monthly or quarterly. Unlike a moving average, exponential smoothing does not require a great deal of historical data. However, it should not be used with data which has more than a moderate amount of up or down trend.

When using exponential smoothing, a smoothing factor is chosen which affects the sensitivity of the average much the same way as the length of the standard moving average period. The correspondence between the two techniques can be represented by the formula:

$$\alpha = \frac{2}{n+1}$$

where $\alpha$ is the exponential smoothing factor (with values from 0 to 1) and $n$ is the length of the standard moving average. As the equation shows, the longer the moving average period, the smaller the equivalent and the less sensitive the average becomes to fluctuations in current values.

Forecasting with exponential smoothing involves selecting the best smoothing factor based on historical data and then using the factor for updating subsequent data and forecasting. This procedure uses the following HP 12C program:

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f [P/R]</td>
<td></td>
</tr>
<tr>
<td>F CLEAR PRGM</td>
<td>00-</td>
</tr>
<tr>
<td>ENTER</td>
<td>01- 36</td>
</tr>
<tr>
<td>ENTER</td>
<td>02- 36</td>
</tr>
<tr>
<td>RCL 6</td>
<td>03- 45 6</td>
</tr>
<tr>
<td>-</td>
<td>04- 30</td>
</tr>
<tr>
<td>ENTER</td>
<td>05- 36</td>
</tr>
<tr>
<td>x</td>
<td>06- 20</td>
</tr>
<tr>
<td>STO + 4</td>
<td>07- 44 40 4</td>
</tr>
<tr>
<td>g LSTX</td>
<td>08- 43 36</td>
</tr>
<tr>
<td>R/S</td>
<td>09- 31</td>
</tr>
<tr>
<td>R+</td>
<td>10- 33</td>
</tr>
<tr>
<td>R-</td>
<td>11- 33</td>
</tr>
<tr>
<td>RCL 0</td>
<td>12- 45 0</td>
</tr>
<tr>
<td>x</td>
<td>13- 20</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>1</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td><strong>x</strong></td>
<td><strong>16</strong></td>
</tr>
<tr>
<td><strong>+</strong></td>
<td><strong>17</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td><strong>CHS</strong></td>
<td><strong>19</strong></td>
</tr>
<tr>
<td><strong>x²y</strong></td>
<td><strong>20</strong></td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td><strong>+</strong></td>
<td><strong>22</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>0</strong></td>
</tr>
<tr>
<td><strong>x</strong></td>
<td><strong>24</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>1</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td><strong>x</strong></td>
<td><strong>27</strong></td>
</tr>
<tr>
<td><strong>+</strong></td>
<td><strong>28</strong></td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>1</strong></td>
</tr>
<tr>
<td><strong>x</strong></td>
<td><strong>31</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>0</strong></td>
</tr>
<tr>
<td><strong>+</strong></td>
<td><strong>33</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td><strong>+</strong></td>
<td><strong>35</strong></td>
</tr>
<tr>
<td><strong>STO</strong></td>
<td></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>0</strong></td>
</tr>
<tr>
<td><strong>+</strong></td>
<td><strong>39</strong></td>
</tr>
<tr>
<td><strong>RCL</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td><strong>+</strong></td>
<td><strong>41</strong></td>
</tr>
</tbody>
</table>

89
Selecting the "best" smoothing constant ($\alpha$):

1. Key in the program and press $f$ CLEAR REG.
2. Key in the number 1 and press ENTER.
4. Key in the first historical value ($X_1$) and press $STO$ 2.
5. Key in the second historical value ($X_2$) and press $STO$ 6 $R/S$. The result is the error between the forecast value ($\hat{X}_{t+1}$) and the true value ($X_{t+1}$)
6. Press $R/S$; the display shows the next forecast ($\hat{X}_{t+2}$).
7. Optional: Press $RCL$ 5 to display the smoothed estimate of current demand.
8. Continue steps 5 and 6 for $X_3$, $X_4$, ..., $X_n$ until all historical values have been entered. When doing step 5 merely key in the value and press $R/S$ (do not press $STO$ 6).
9. Press $RCL$ 4. This value represents the cumulative forecasting error ($\sum e^2$). Record the value and the following additional values: press $RCL$ 0 ($\alpha$), $RCL$ 2 (smoothed average $S_{t-1}$), $RCL$ 3 (trend $T_{t-1}$) and $RCL$ 6 (forecast $\hat{X}_{t+1}$).
10. Press $f$ CLEAR REG.
11. Repeat steps 2 through 10 until a "best" $\alpha$ is selected based on the lowest cumulative forecasting error (Register 4).

Forecasting:
1. Key in the number 1 and press **ENTER**

2. Key in the selected and press **STO B** STO 1.

3. From the selection routing or from a previous forecast:
   - Key in the smoothed average $S_{t-1}$ and press **STO 2**.
   - Key in the trend $T_{t-1}$ and press **STO 3**.
   - Key in the forecast $t+1$ and press **STO 6**.

4. Key in the current data value and press **R/S**. The output is the error in forecasting the value just entered.

5. Press **R/S**. The displayed value represents the forecast for the next period.

6. Record the following values: **RCL 0** ($\alpha$), **RCL 2** ($S_{t-1}$), **RCL 3** ($T_{t-1}$) and **RCL 6** ($D_{t+1}$) for use as initial values in the next forecast. You may also wish to record **RCL 5** ($D_t$).

7. Repeat steps 4, 5, and 6 for the next forecast if available.

**Example:** Select the best smoothing constant based on sales (in thousands of dollars) of 22, 23, 23, 25, 23, 27, 25. Given the current sales in month 8 of 26, forecast the following month. Select the smoothing constant ($\alpha$):

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CLEAR</strong> REG</td>
<td>0.00</td>
</tr>
<tr>
<td>1 <strong>ENTER</strong></td>
<td>1.00</td>
</tr>
<tr>
<td>5 <strong>STO</strong> 0</td>
<td>0.50</td>
</tr>
<tr>
<td><strong>STO</strong> 1</td>
<td>0.50</td>
</tr>
<tr>
<td>22 <strong>STO</strong> 2</td>
<td>22.00</td>
</tr>
<tr>
<td>23 <strong>STO</strong> 6 <strong>R/S</strong></td>
<td>0.00</td>
</tr>
<tr>
<td><strong>R/S</strong></td>
<td>23.00</td>
</tr>
<tr>
<td>23 <strong>R/S</strong> <strong>R/S</strong></td>
<td>23.25</td>
</tr>
<tr>
<td>25 <strong>R/S</strong> <strong>R/S</strong></td>
<td>25.25</td>
</tr>
<tr>
<td>23 <strong>R/S</strong> <strong>R/S</strong></td>
<td>23.69</td>
</tr>
<tr>
<td>27 <strong>R/S</strong> <strong>R/S</strong></td>
<td>27.13</td>
</tr>
<tr>
<td>25 <strong>R/S</strong> <strong>R/S</strong></td>
<td>25.95</td>
</tr>
</tbody>
</table>
The procedure is repeated for several $\alpha$'s.

<table>
<thead>
<tr>
<th>Smoothing Constant ($\alpha$)</th>
<th>0.5</th>
<th>0.1</th>
<th>0.25</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Error ($\Sigma e^2$)</td>
<td>23.61</td>
<td>25.14</td>
<td>17.01</td>
<td>18.03</td>
</tr>
</tbody>
</table>

For the selected $\alpha = 0.25$

$S_{t+1} = 24.28$

$T_{t-1} = 0.34$

$D_{t+1} = 25.64$

Forecasting:

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{RCL} \ 4$</td>
<td>23.61</td>
</tr>
<tr>
<td>$\text{RCL} \ 0$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\text{RCL} \ 2$</td>
<td>25.11</td>
</tr>
<tr>
<td>$\text{RCL} \ 3$</td>
<td>0.42</td>
</tr>
<tr>
<td>$\text{RCL} \ 6$</td>
<td>25.95</td>
</tr>
</tbody>
</table>

The procedure is repeated for several $\alpha$'s.

Note: At least 4 periods of current data should be entered before forecasting is attempted.
Pricing Calculations

Markup and Margin Calculations

Sales work often involves calculating the various relations between markup, margin, selling price and costs. Markup is defined as the difference between selling price and cost, divided by the cost. Margin is defined as the difference between selling price and cost, divided by selling price. In other words, markup is based on cost and margin is based on selling price.

The following keystroke sequences are given to readily make these calculations on the HP-12C.

<table>
<thead>
<tr>
<th>CALCULATE</th>
<th>GIVEN</th>
<th>KEYSTROKES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling Price</td>
<td>Cost &amp; Markup</td>
<td>Key in cost, ENTER, key in markup (in %), % +.</td>
</tr>
<tr>
<td>Selling Price</td>
<td>Cost &amp; Margin</td>
<td>Key in cost, ENTER 1 ENTER, key in margin (in %).% - +.</td>
</tr>
<tr>
<td>Cost</td>
<td>Selling Price &amp; Markup</td>
<td>Key in selling price, ENTER 1 ENTER, key in markup (in %), % + - +.</td>
</tr>
<tr>
<td>Cost</td>
<td>Selling Price &amp; Margin</td>
<td>Key in selling price, ENTER 1 ENTER, key in margin (in %), % - +.</td>
</tr>
<tr>
<td>Markup</td>
<td>Cost and Selling Price</td>
<td>Key in cost, ENTER, key in selling price, Δ%.</td>
</tr>
<tr>
<td>Markup</td>
<td>Margin</td>
<td>Key in margin, ENTER ENTER 1 x¿y, %, - +.</td>
</tr>
<tr>
<td>Margin</td>
<td>Selling Price &amp; Cost</td>
<td>Key in selling price, ENTER, key in cost, Δ% CHS.</td>
</tr>
<tr>
<td>Margin</td>
<td>Markup</td>
<td>Key in markup, ENTER ENTER 1 x¿y, %, + - +.</td>
</tr>
</tbody>
</table>

Example 1: If the cost of an item is $160 and the margin is 20%, what is the selling price? What is the markup?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>160 ENTER</td>
<td>160.00 Cost.</td>
</tr>
<tr>
<td>1 ENTER 20</td>
<td>20.00 Margin (%).</td>
</tr>
<tr>
<td>% - +</td>
<td>200.00 Selling price.</td>
</tr>
<tr>
<td>20 ENTER ENTER</td>
<td>20.00</td>
</tr>
<tr>
<td>1 x¿y% - +</td>
<td>25.00 Markup (%).</td>
</tr>
</tbody>
</table>
Example 2: If an item sells for $21.00 and has a markup of 50%, what is its cost? What is the margin?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 ENTER</td>
<td>21.00</td>
</tr>
<tr>
<td>1 ENTER 50</td>
<td>50.00</td>
</tr>
<tr>
<td>% + +</td>
<td>14.00</td>
</tr>
<tr>
<td>50 ENTER ENTER</td>
<td>50.00</td>
</tr>
<tr>
<td>1 x3y % + +</td>
<td>33.33</td>
</tr>
</tbody>
</table>

The following HP 12C program may be helpful for repetitive calculations of selling price and costs as well as conversions between markup and margin.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td></td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>00-</td>
</tr>
<tr>
<td>ENTER</td>
<td>01- 36</td>
</tr>
<tr>
<td>g GTO 04</td>
<td>02-43, 33</td>
</tr>
<tr>
<td>CHS</td>
<td>03- 16</td>
</tr>
<tr>
<td>x3y % + +</td>
<td>05- 34</td>
</tr>
<tr>
<td>% + +</td>
<td>06- 25</td>
</tr>
<tr>
<td>+ +</td>
<td>07- 40</td>
</tr>
<tr>
<td>R/S</td>
<td>09- 31</td>
</tr>
<tr>
<td>g LSTX</td>
<td>10- 43 36</td>
</tr>
<tr>
<td>x</td>
<td>11- 20</td>
</tr>
<tr>
<td>g LSTX</td>
<td>12- 43 36</td>
</tr>
<tr>
<td>x</td>
<td>13- 20</td>
</tr>
<tr>
<td>g GTO</td>
<td>14-43, 33 00</td>
</tr>
<tr>
<td>f P/R</td>
<td></td>
</tr>
</tbody>
</table>

**Registers**

- n: Unused
- i: Unused
- PV: Unused
- PMT: Unused
1. Key in program.

2. To calculate selling price, given the markup, key in the cost, press \[ \text{ENTER} \text{ } g \text{ GTO } 00 \text{ R/S R/S} \] key in the markup and press \[ g \text{ GTO } 00 \text{ R/S R/S} \]

3. To calculate cost, given the markup, key in the selling price, press \[ \text{ENTER} \text{ } g \text{ GTO } 00 \text{ R/S R/S} \] key in the markup and press \[ g \text{ GTO } 00 \text{ R/S R/S} \]

4. To calculate selling price, given the margin, key in the cost, press \[ \text{ENTER} \text{ } g \text{ GTO } 03 \text{ R/S R/S} \] key in the margin and press \[ g \text{ GTO } 03 \text{ R/S R/S} \]

5. To calculate cost given the margin, key in the selling price, press \[ \text{ENTER} \text{ } g \text{ GTO } 03 \text{ R/S R/S} \] key in the margin and press \[ g \text{ GTO } 03 \text{ R/S R/S} \]

6. To calculate markup from the margin, key in the cost and press \[ \text{ENTER} \text{ } g \text{ GTO } 03 \text{ R/S R/S} \]

7. To calculate margin from the markup, key in the markup and press \[ \text{ENTER} \text{ } g \text{ GTO } 00 \text{ R/S R/S} \]

Example: Find the cost of an item selling for $38.00 with a margin of 30%. What is the markup on the item? If the markup is raised to 50%, what will the selling price be?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>38 ENTER</td>
<td>38.00</td>
</tr>
<tr>
<td>30 g GTO 03</td>
<td>30.00</td>
</tr>
<tr>
<td>R/S R/S</td>
<td>26.60</td>
</tr>
<tr>
<td>30 ENTER g GTO 03</td>
<td>42.86</td>
</tr>
<tr>
<td>R/S R/S</td>
<td>26.60</td>
</tr>
<tr>
<td>50 g GTO 00</td>
<td>39.90</td>
</tr>
</tbody>
</table>

Calculations of List and Net prices With Discounts

If it often useful to be able to quickly calculate list or net price when the other price and a series of discount rates are known. Alternatively, if the
list and new and several discounts are known it may be desirable to calculate
a missing discount. The following series of keystrokes may be used:

1. Key in 1, press \[ \text{ENTER} \text{ ENTER} \text{ ENTER} \text{ STO} 1. \]
2. Key in the first discount (as a percentage) and press \[ \% - \text{ STO} \times 1 \text{ R+}. \]
3. Repeat step 2 for each of the remaining known discount rates.
4. To calculate the list price, key in the net price and press \[ \text{RCL} 1 + . \]
5. To calculate the net price, key in the list price and press \[ \text{RCL} 1 \times . \]
6. To calculate an unknown discount rate, immediately after doing step 3
   (display should show 1.00), key in the net price, press \[ \text{ENTER} \] and key
   in the list price.
7. Press \[ \text{RCL} 1 \div 100 \times . \]

**Example:** The list price of an item is $3.28 and the net price is $1.45. Two
of the discount rates are 48% and 5%. What is the third discount rate?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 [ \text{ENTER} \text{ ENTER} ]</td>
<td>1.00</td>
</tr>
<tr>
<td>STO 1</td>
<td>1.00</td>
</tr>
<tr>
<td>48 [ % - \text{ STO} \times 1 \text{ R+} ]</td>
<td>1.00</td>
</tr>
<tr>
<td>5 [ % - \text{ STO} \times 1 \text{ R+} ]</td>
<td>1.00</td>
</tr>
<tr>
<td>1.45 [ \text{ENTER} ]</td>
<td>0.49</td>
</tr>
<tr>
<td>3.28 [ \text{RCL} 1 ]</td>
<td>0.49</td>
</tr>
<tr>
<td>[ \times \div 100 \times ]</td>
<td>10.51 3rd discount rate (%).</td>
</tr>
</tbody>
</table>

The following program for the HP 12C will be helpful in performing the
calculations:

<table>
<thead>
<tr>
<th>KEYS &amp; DISPLAY</th>
<th>KEYS &amp; DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ f \text{ P/R} ]</td>
<td>00-</td>
</tr>
<tr>
<td>[ f \text{ CLEAR PRGM} ]</td>
<td>01- 1</td>
</tr>
<tr>
<td>[ x \equiv y \times 100 ]</td>
<td>02- 34</td>
</tr>
<tr>
<td>%</td>
<td>03- 25</td>
</tr>
</tbody>
</table>
1. Key in the program.
2. Key in 1 and press \( \text{STO} \) 1.
3. Key in the first discount rate (as a percentage) and press \( \text{R/S} \).
4. Repeat step 2 for each of the remaining discount rates.
5. To calculate the list price, key in the net price and press \( \text{RCL} \) 1 + .
6. To calculate the net price, key in the list price and press \( \text{RCL} \) 1 .
7. To calculate the unknown discount rate, key in the net price, press \( \text{ENTER} \), key in the list price and press \( \text{g} \) \( \text{GTO} \) 07 \( \text{R/S} \).

**Example:** Calculate the unknown discount rate for the previous example. If the list price is now raised to $3.75 what is the new net price?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>04-30</td>
</tr>
<tr>
<td>( \text{STO} ) x 1</td>
<td>05-44 20 1</td>
</tr>
<tr>
<td>( \text{g} ) ( \text{GTO} ) 00</td>
<td>06-43, 33 00</td>
</tr>
<tr>
<td>( \text{RCL} ) 1</td>
<td>07-45 1</td>
</tr>
<tr>
<td>x</td>
<td>08-20</td>
</tr>
<tr>
<td>÷</td>
<td>09-10</td>
</tr>
<tr>
<td>i</td>
<td>10-1</td>
</tr>
<tr>
<td>x( \text{z/y} )</td>
<td>11-34</td>
</tr>
<tr>
<td>( \text{EEX} )</td>
<td>12-30</td>
</tr>
<tr>
<td>2</td>
<td>13-26</td>
</tr>
<tr>
<td>x</td>
<td>14-2</td>
</tr>
<tr>
<td>( \text{g} ) ( \text{GTO} ) 00</td>
<td>15-20</td>
</tr>
<tr>
<td>( \text{R} ) ( \text{P/R} )</td>
<td>16-43, 33 00</td>
</tr>
</tbody>
</table>

**REGISTERS**

- n: Unused
- i: Unused
- PV: Unused
- PMT: Unused
- FV: Unused
- \( R_0 \): Unused
- \( R_1 \): \( R_1 \), \( R_2 \), \( R_3 \), \( R_4 \), \( R_5 \), \( R_6 \), \( R_7 \): Unused
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>STO 1</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>R/S</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>R/S</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>1.45</td>
<td>ENTER</td>
<td>10.51</td>
<td>3rd discount rate (%)</td>
</tr>
<tr>
<td>3.28</td>
<td>0</td>
<td>STO 07</td>
<td>R/S</td>
</tr>
<tr>
<td>0.89</td>
<td>R/S</td>
<td>Include 3rd discount rate in calculation</td>
<td></td>
</tr>
<tr>
<td>3.75</td>
<td>RCL 1</td>
<td>x</td>
<td>1.66</td>
</tr>
</tbody>
</table>
Statistics

Curve Fitting

Exponential Curve Fit

Using the \( \ln \) function of the HP-12C, a least squares exponential curve fit may be easily calculated according to the equation \( y = Ae^{Bx} \). The exponential curve fitting technique is often used to determine the growth rate of a variable such as a stock’s value over time, when it is suspected that the performance is non-linear. The value for \( B \) is the decimal value of the continuous growth rate. For instance, assume after keying in several end-of-month price quotes for a particular stock it is determined that the value of \( B \) is 0.10. This means that over the measured growth period the stock has experienced a 10% continuous growth rate.

If \( B > 0 \), you will have a growth curve. If \( B < 0 \), you will have a decay curve. Examples of these are given below.

The procedure is as follows:

1. Press \( \boxed{\text{F CLEAR REG}} \).
2. For each input pair of values, key in the y-value and press \( \boxed{g \ln} \), key in the corresponding x-value and press \( \boxed{\sum +} \).
3. After all data pairs are input, press \( \boxed{\hat{y} \sum x \sum y} \) to obtain the correlation coefficient (between \( \ln y \) and \( x \)).
4. Press \( \boxed{1 0} \) \( \boxed{g \hat{y} \sum x} \boxed{e^x} \) \( \boxed{g \hat{y} \sum x} \boxed{e^x} \) to obtain \( A \) in the equation above.
5. Press [align right x<>y] to obtain B.

6. Press [align right e^x] to obtain the effective growth rate (as a decimal).

7. To make a y-estimate, key in the x-value and press [align right y^x r g e^x]

Example 1: A stock's price in history is listed below. What effective growth rate does this represent? If the stock continues this growth rate, what is the price projected to be at the end of 1982 (year 7)?

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976(1)</td>
<td>45</td>
</tr>
<tr>
<td>1977(2)</td>
<td>51.5</td>
</tr>
<tr>
<td>1978(3)</td>
<td>53.75</td>
</tr>
<tr>
<td>1979(4)</td>
<td>80</td>
</tr>
<tr>
<td>1980(5)</td>
<td>122.5</td>
</tr>
<tr>
<td>1981(6)</td>
<td>210</td>
</tr>
<tr>
<td>1982(7)</td>
<td>?</td>
</tr>
</tbody>
</table>

Keystrokes | Display |
---|---------|
[align left f] CLEAR REG | 1.00 First data pair input. |
45 [align left g] LN 1 [align right ] [align right ] [align right ] Sum+ | 2.00 Second data pair input. |
51.5 [align left g] LN 2 [align right ] [align right ] [align right ] Sum+ | 3.00 Third data pair input. |
53.75 [align left g] LN 2 [align right ] [align right ] [align right ] Sum+ | 4.00 Fourth data pair input. |
80 [align left g] LN 2 [align right ] [align right ] [align right ] Sum+ | 5.00 Fifth data pair input. |
122.5 [align left g] LN 2 [align right ] [align right ] [align right ] Sum+ | 6.00 Sixth data pair input. |
[align left g] y^x r [align right ] x<>y | 0.95 Correlation coefficient (between ln y and x). |
1 [align left g] y^x r [align left g] e^x | 27.34 A |
0 [align left g] y^x r [align left g] e^x | 0.31 B |
[align left x<>y] R+ [align right ] [align right ] [align right ] g LN | 0.31 |
[align left g] e^x 1 [align right ] [align right ] [align right ] [align right ] - | 0.36 Effective growth rate. |
7 [align left g] y^x r [align left g] e^x | 232.35 Projected price at end of year / (1982). |

For repeated use of this routine, the following HP-12C program will be useful.
<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
<th>Key Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEAR</td>
<td>Clear</td>
<td>00-</td>
</tr>
<tr>
<td>PRGM</td>
<td>Program</td>
<td></td>
</tr>
<tr>
<td>x &gt; y</td>
<td>Greater than</td>
<td>01- 34</td>
</tr>
<tr>
<td>LN</td>
<td>Natural log</td>
<td>02- 43 23</td>
</tr>
<tr>
<td>x &gt; y</td>
<td>Greater than</td>
<td>03- 34</td>
</tr>
<tr>
<td>+</td>
<td>Add</td>
<td>04- 49</td>
</tr>
<tr>
<td>GTO 00</td>
<td>Goto 00</td>
<td>05- 43, 33 00</td>
</tr>
<tr>
<td>y^r</td>
<td>y to the power of x</td>
<td>06- 43 2</td>
</tr>
<tr>
<td>x &gt; y</td>
<td>Greater than</td>
<td>07- 34</td>
</tr>
<tr>
<td>R/S</td>
<td>Roll/Swap</td>
<td>08- 31</td>
</tr>
<tr>
<td>1</td>
<td>Number 1</td>
<td>09- 1</td>
</tr>
<tr>
<td>y^r</td>
<td>y to the power of x</td>
<td>10- 43 2</td>
</tr>
<tr>
<td>e^x</td>
<td>Exponential</td>
<td>11- 43 22</td>
</tr>
<tr>
<td>0</td>
<td>Number 0</td>
<td>12- 0</td>
</tr>
<tr>
<td>y^r</td>
<td>y to the power of x</td>
<td>13- 43 2</td>
</tr>
<tr>
<td>e^x</td>
<td>Exponential</td>
<td>14- 43 22</td>
</tr>
<tr>
<td>R/S</td>
<td>Roll/Swap</td>
<td>15- 31</td>
</tr>
<tr>
<td>x &gt; y</td>
<td>Greater than</td>
<td>16- 34</td>
</tr>
<tr>
<td>R↓</td>
<td>Roll Down</td>
<td>17- 33</td>
</tr>
<tr>
<td>+</td>
<td>Add</td>
<td>18- 10</td>
</tr>
<tr>
<td>LN</td>
<td>Natural log</td>
<td>19- 43 23</td>
</tr>
<tr>
<td>R/S</td>
<td>Roll/Swap</td>
<td>20- 31</td>
</tr>
<tr>
<td>e^x</td>
<td>Exponential</td>
<td>21- 43 22</td>
</tr>
<tr>
<td>1</td>
<td>Number 1</td>
<td>22- 1</td>
</tr>
<tr>
<td>−</td>
<td>Subtract</td>
<td>23- 30</td>
</tr>
<tr>
<td>R/S</td>
<td>Roll/Swap</td>
<td>24- 31</td>
</tr>
<tr>
<td>y^r</td>
<td>y to the power of x</td>
<td>25- 43 2</td>
</tr>
<tr>
<td>e^x</td>
<td>Exponential</td>
<td>26- 43 22</td>
</tr>
<tr>
<td>GTO 00</td>
<td>Goto 00</td>
<td>27- 43, 33 00</td>
</tr>
</tbody>
</table>

101
1. Key in the program and press \( f \) CLEAR REG.

2. For each input pair of values, key in the y-value and press \( \text{ENTER} \), key in the corresponding x-value and press \( \text{R/S} \).

3. After all data pairs are input, press \( g \) GTO 06 R/S to obtain the correlation coefficient (between \( \ln y \) and \( x \)).

4. Press \( \text{R/S} \) to obtain \( A \).

5. Press \( \text{R/S} \) to obtain \( B \).

6. Press \( \text{R/S} \) to obtain the effective growth rate as a decimal.

7. To make a y-estimate, key in the x-value and press \( \text{R/S} \). For subsequent estimates, key in the x-value and press \( g \) GTO 25 R/S.

8. For a different set of data, press \( f \) CLEAR REG and go to step 2.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) CLEAR REG</td>
<td></td>
</tr>
<tr>
<td>45 ENTER 1 R/S</td>
<td>1.00 First data pair input.</td>
</tr>
<tr>
<td>51.5 ENTER 2 R/S</td>
<td>2.00 Second data pair input.</td>
</tr>
<tr>
<td>53.75 ENTER 3 R/S</td>
<td>3.00 Third data pair input.</td>
</tr>
<tr>
<td>80 ENTER 4 R/S</td>
<td>4.00 Fourth data pair input.</td>
</tr>
<tr>
<td>122.5 ENTER 5 R/S</td>
<td>5.00 Fifth data pair input.</td>
</tr>
<tr>
<td>210 ENTER 6 R/S</td>
<td>6.00 Sixth data pair input.</td>
</tr>
<tr>
<td>( g ) GTO 06 R/S</td>
<td>0.95 Correlation coefficient (between ( \ln y ) and ( x )).</td>
</tr>
</tbody>
</table>
Logarithmic Curve Fit

If your data does not fit a line or an exponential curve, try the following logarithmic curve fit. This is calculated according to the equation \( y = A + B \ln(x) \), and all \( x \) values must be positive.

A typical logarithmic curve is shown below.

<table>
<thead>
<tr>
<th>R/S</th>
<th>27.34</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R/S</td>
<td>0.31</td>
<td>( B )</td>
</tr>
<tr>
<td>R/S</td>
<td>0.36</td>
<td>Effective growth rate.</td>
</tr>
<tr>
<td>7 R/S</td>
<td>232.35</td>
<td>Projected price at the end of year 7 (1982).</td>
</tr>
</tbody>
</table>

The procedure is as follows:
1. Press \( \text{f CLEAR} \) \( \text{REG} \).
2. Key in the first \( y \)-value and press \( \text{ENTER} \). Key in the first \( x \)-value and press \( \text{g LN} \) \( \Sigma^+ \). Repeat this step for each data pair.
3. After all data pairs are input, press \( \text{g} \) \( \hat{y}_r \) \( x \hat{y}_r \) to obtain the correlation coefficient (between \( y \) and \( \ln x \)).
4. Press \( 1 \) \( \text{g} \) \( \hat{y}_r \) \( 0 \) \( \text{g} \) \( \hat{y}_r \) to obtain \( A \) in the equation above.
5. Press \( x \hat{y}_r \) \( R \downarrow \) \( - \) to obtain \( B \).
6. To make a \( y \)-estimate, key in the \( x \)-value and press \( \text{g LN} \) \( \text{g} \) \( \hat{y}_r \).

Example 1: A manufacturer observes declining sales of a soon-to-be obsolete product, of which there were originally 10,000 units in inventory. The cumulative sales figures over a number of months, given below, may be fit by a logarithmic cure of the form \( y = A + B \ln x \), where \( y \) represents cumulative sales in units and \( x \) the number of months since the beginning. How many units will be sold by the end of eighth months?

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Sales (units)</td>
<td>1431</td>
<td>3506</td>
<td>5177</td>
<td>6658</td>
<td>7810</td>
<td>8592</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{f CLEAR} ) ( \text{REG} )</td>
<td></td>
</tr>
<tr>
<td>1431 ( \text{ENTER} )</td>
<td>1.00 First pair data input.</td>
</tr>
<tr>
<td>1 ( \text{g LN} ) ( \Sigma^+ )</td>
<td></td>
</tr>
<tr>
<td>3506 ( \text{ENTER} )</td>
<td>2.00 Second pair data input.</td>
</tr>
<tr>
<td>2 ( \text{g LN} ) ( \Sigma^+ )</td>
<td></td>
</tr>
<tr>
<td>5177 ( \text{ENTER} )</td>
<td>3.00 Third pair data input.</td>
</tr>
<tr>
<td>3 ( \text{g LN} ) ( \Sigma^+ )</td>
<td></td>
</tr>
<tr>
<td>6658 ( \text{ENTER} )</td>
<td>4.00 Forth pair data input.</td>
</tr>
<tr>
<td>4 ( \text{g LN} ) ( \Sigma^+ )</td>
<td></td>
</tr>
<tr>
<td>7810 ( \text{ENTER} )</td>
<td>5.00 Fifth pair data input.</td>
</tr>
<tr>
<td>5 ( \text{g LN} ) ( \Sigma^+ )</td>
<td></td>
</tr>
<tr>
<td>8592 ( \text{ENTER} )</td>
<td>6.00 Sixth pair data input.</td>
</tr>
<tr>
<td>6 ( \text{g LN} ) ( \Sigma^+ )</td>
<td></td>
</tr>
</tbody>
</table>
Another method of analysis is the power curve or geometric curve. The equation of the power curve is \( y = A x^B \), and the values for \( A \) and \( B \) are computed by calculations similar to linear regression. Some examples of power curves are shown below.

The following keystrokes fit a power curve according to the equation \( \ln y = \ln A + B \ln x \):  

1. Press \( \text{CLEAR} \).  
2. Key in the first \( y \)-value and press \( g \ \ln \). Key in the first \( x \)-value and press \( g \ \ln \ \Sigma^+ \). Repeat this step for all data pairs.  
3. Press \( g \ \hat{y}, \ r \ x \ \Sigma y \) to obtain the correlation coefficient (between \( \ln y \) and \( \ln x \)).  
4. Press \( 0 \ \hat{y}, \ r \ \ g \ e^x \) to obtain \( A \) in the above equation.  
5. Press \( 1 \ \hat{y}, \ r \ 0 \ g \ \hat{y}, \ r \ x \ \Sigma y \ \text{R}\Rightarrow \) to obtain \( B \).  
6. To make a \( y \)-estimate, key in the \( x \)-value and press \( g \ \ln \ g \ \hat{y}, \ r \ g \ e^x \).

**Example:** If Galileo had wished to investigate quantitatively the relationship between the time (\( t \)) for a falling object to hit the ground and the height (\( h \)) it hasfallen, he might have released a rock from various
levels of the Tower of Pisa (which was leaning even then) and timed its descent by counting his pulse. The following data are measurements Galileo might have made.

<table>
<thead>
<tr>
<th>$t$ (pulses)</th>
<th>2</th>
<th>2.5</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ (feet)</td>
<td>30</td>
<td>50</td>
<td>90</td>
<td>130</td>
<td>150</td>
</tr>
</tbody>
</table>

Find the power curve formulas that best expresses $h$ as a function of $t$ ($h = At^B$).

The formula that best expresses $h$ as a function of $t$ is

$h = 7.72t^{1.99}$

We know, as Galileo did not, that in fact $h$ is proportional to $t^2$.

**Standard Error of the Mean**

The standard error of the mean is a measure of how reliable the mean of a sample ($\bar{x}$) is as an estimator of the mean of the population from which the sample was drawn.

To calculate the standard error of the mean:
1. Press \[ f \text{ CLEAR REG} \].

2. If you are summing one set of numbers, key in the first number and press \[ \Sigma + \]. Continue until you have entered all of the values.

3. If you are summing two sets of numbers, key in the \( y \)-value and press \[ \text{ENTER} \], key in the \( x \)-value and press \[ \Sigma + \]. Continue until you have entered all of the values.

4. Press \[ g \] \( \bar{x} \) to obtain the mean of the \( x \)-values.

5. Press \[ g \] \( s \) \( \sigma \) \( \Sigma \) \( 1 \) \( \sqrt{\sigma} \) \( \div \) to obtain the standard error of the mean of the \( x \)-values.

6. Alternatively, press \[ g \] \( s \) \( x \) \( y \) \( \Sigma \) \( 1 \) \( \sqrt{\sigma} \) \( \div \) to obtain the standard error for the mean of the \( y \)-values.

**Example:** A sample of 6 one-bedroom apartment rentals reveals that one rents for $190 per month unfurnished, one rents for $200 per month, two rent for $205 per month, one rents for $216 per month, and one rents for $220 per month. What are the mean monthly rental and the standard deviation? What is the standard error of the mean?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f \text{ CLEAR REG} )</td>
<td>6.00 Total number of inputs.</td>
</tr>
<tr>
<td>190 ( \Sigma + ) 200 ( \Sigma + )</td>
<td>206.00 Average monthly rent.</td>
</tr>
<tr>
<td>205 ( \Sigma + ) 205 ( \Sigma + ) 216 ( \Sigma + )</td>
<td>10.86 Standard deviation.</td>
</tr>
<tr>
<td>220 ( \Sigma + )</td>
<td>4.43 Standard error of the mean.</td>
</tr>
</tbody>
</table>

**Mean, Standard Deviation, Standard Error for Grouped Data**

Grouped data are presented in frequency distributions to save time and effort in writing down (or entering) each observation individually. Given a set of data points

\[ x_1, x_2, \ldots, x_n \]

with respective frequencies

\[ f_1, f_2, \ldots, f_n \]
this procedure computes the mean, standard deviation, and standard error of the mean.

1. Press \( \text{f} \) CLEAR REG.
2. Key in the first value and press ENTER ENTER.
3. Key in the respective frequency and press \( \text{STO} + \) \( \times \) \( \Sigma^+ \). The display shows the number of data points entered.
4. Repeat steps 2 and 3 for each data point.
5. To calculate the mean (average) press \( \text{RCL} \) \( 0 \) \( \text{STO} \) \( 1 \) \( \text{RCL} \) \( 0 \) \( \text{STO} \) \( 3 \).
6. Press \( \text{g} \) \( \overline{X} \) to find the standard deviation.
7. Press \( \text{RCL} \) \( 0 \) \( \text{g} \) \( \sqrt{\chi^2} \) \( + \) to find the standard error of the mean.

**Example 1:** A survey of 266 one-bedroom apartment rentals reveals that 54 rent for $190 a month unfurnished, 32 rent for $195 per month, 88 rent for $200 per month, and 92 rent for 206 per month. What are the average monthly rental, the standard deviation, and the standard error of the mean?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{f} ) CLEAR REG</td>
<td></td>
</tr>
<tr>
<td>190 ENTER ENTER ENTER</td>
<td>1.00 First data pair entered.</td>
</tr>
<tr>
<td>195 ENTER ENTER ENTER</td>
<td>2.00 Second data pair entered.</td>
</tr>
<tr>
<td>32 ENTER ENTER ENTER</td>
<td>3.00 Third data pair entered.</td>
</tr>
<tr>
<td>200 ENTER ENTER ENTER</td>
<td>4.00 Fourth data pair entered.</td>
</tr>
<tr>
<td>88 ENTER ENTER ENTER</td>
<td></td>
</tr>
<tr>
<td>206 ENTER ENTER ENTER</td>
<td></td>
</tr>
<tr>
<td>92 ENTER ENTER ENTER</td>
<td></td>
</tr>
<tr>
<td>( \text{RCL} ) ( 0 ) ( \text{STO} ) ( 1 ) ( \text{RCL} ) ( 5 )</td>
<td>199.44 Average monthly rent.</td>
</tr>
<tr>
<td>( \text{STO} ) ( 3 ) ( \text{g} ) ( \overline{X} )</td>
<td></td>
</tr>
<tr>
<td>( \text{g} ) ( \sqrt{\chi^2} ) ( + )</td>
<td>5.97 Standard deviation.</td>
</tr>
<tr>
<td>( \text{RCL} ) ( 0 ) ( \text{g} ) ( \sqrt{\chi^2} ) ( + )</td>
<td>0.37 Standard error of the mean.</td>
</tr>
</tbody>
</table>

Use the following HP-12C program for the previous example:
1. Key in the program.
2. Press \( f \) [CLEAR] [REG]
3. Key in the first value and press [ENTER] [ENTER]
4. Key in the respective frequency and press \[ R/S \] The display shows the number of data points entered.
5. Repeat steps 3 and 4 for each data point.
6. To calculate the mean, press \[ g \] GTO 05 \[ R/S \].
7. Press \[ R/S \] to find the standard deviation.
8. Press \[ R/S \] to find the standard error of the mean.
9. For a new case, go to step 2.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEAR R/S</td>
<td>1.00</td>
</tr>
<tr>
<td>ENTER ENTER</td>
<td>2.00</td>
</tr>
<tr>
<td>R/S</td>
<td>3.00</td>
</tr>
<tr>
<td>ENTER ENTER</td>
<td>4.00</td>
</tr>
<tr>
<td>R/S</td>
<td>199.44</td>
</tr>
<tr>
<td>GTO 05 R/S</td>
<td>5.97</td>
</tr>
<tr>
<td>R/S</td>
<td>0.37</td>
</tr>
</tbody>
</table>

**Chi-Square Statistics**

The chi-square statistic is a measure of the goodness of fit between two sets of frequencies. It is used to test whether a set of observed frequencies differs from a set of expected frequencies sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies \( (O) \) and the expected frequencies \( (E) \) are significant, or whether they may reasonable be attributed to chance. The formula generally used is:
If there is a close agreement between the observed and expected frequencies, \( x^2 \) will be small. If the agreement is poor, \( x^2 \) will be large.

The following keystrokes calculate the \( x^2 \) statistic:

1. Press \( \boxed{\text{f CLEAR REG}} \).
2. Key in the first \( O_i \) value and press \( \boxed{\text{ENTER}} \).
3. Key in the first \( E_i \) value and press \( \boxed{\text{STO 0 ENTER x RCL 0 +}} \).
4. Repeat steps 2 and 3 for all data pairs. The \( x^2 \) value is displayed.

**Example 1:** A suspect die from a Las Vegas casino is brought to an independent testing firm to determine its bias, if any. The die is tossed 120 times and the following results obtained.

<table>
<thead>
<tr>
<th>Number</th>
<th>Observed Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
</tr>
</tbody>
</table>

The expected frequency = 120 throws / 6 sides, or \( E = 20 \) for each number, 1 thru 6. (Since \( E \) is a constant in this example, there is no need to store it in \( R_0 \) each time.)

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \boxed{\text{f CLEAR REG}} )</td>
<td></td>
</tr>
<tr>
<td>( 25 \boxed{\text{ENTER}} )</td>
<td></td>
</tr>
<tr>
<td>( 20 \boxed{\text{STO 0 ENTER x RCL 0 +}} )</td>
<td>1.25</td>
</tr>
<tr>
<td>( 17 \boxed{\text{ENTER 20 - ENTER x RCL 0 +}} )</td>
<td>1.70</td>
</tr>
</tbody>
</table>
The number of degrees of freedom is \((n-1)\). Since \(n = 6\), the degrees of freedom = 5.

Consulting statistical tables, you look up \(x^2\) to a 0.05 significance level with 5 degrees of freedom, and see that \(x^2_{0.05,5} = 11.07\). Since \(x^2 = 5\) is within 11.07, we may conclude that to a 0.05 significance level (probability = .95), the die is fair.

Try the following HP-12C program with the same example.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00-</td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td></td>
</tr>
<tr>
<td>STO 0</td>
<td>01- 44 00</td>
</tr>
<tr>
<td>-</td>
<td>02- 10</td>
</tr>
<tr>
<td>ENTER</td>
<td>03- 36</td>
</tr>
<tr>
<td>(x )</td>
<td>04- 20</td>
</tr>
<tr>
<td>RCL 0</td>
<td>05- 45 00</td>
</tr>
<tr>
<td>+</td>
<td>06- 10</td>
</tr>
<tr>
<td>+</td>
<td>07- 40</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>08-43, 33 00</td>
</tr>
<tr>
<td>f P/R</td>
<td></td>
</tr>
</tbody>
</table>

**Registers**
- \(n\): Unused
- \(i\): Unused
- \(PV\): Unused
- \(PMT\): Unused
- \(FV\): Unused
- \(R_0\): \(E_i\)
- \(R_1-R_9\): Unused
1. Key in the program.
2. Press \( \text{f} \text{ CLEAR} \text{ REG} \).
3. Key in the first \( O_i \) value and press \( \text{ENTER} \).
4. Key in the first \( E_i \) value and press \( \text{R/S} \).
5. Repeat steps 3 and 4 for all data pairs. The \( x^2 \) value is displayed.
6. For a new case, go to step 2.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{f} \text{ CLEAR} \text{ REG} )</td>
<td></td>
</tr>
<tr>
<td>25 ( \text{ENTER} ) ( \text{R/S} )</td>
<td>1.25</td>
</tr>
<tr>
<td>17 ( \text{ENTER} ) ( \text{R/S} )</td>
<td>1.70</td>
</tr>
<tr>
<td>15 ( \text{ENTER} ) ( \text{R/S} )</td>
<td>2.95</td>
</tr>
<tr>
<td>23 ( \text{ENTER} ) ( \text{R/S} )</td>
<td>3.40</td>
</tr>
<tr>
<td>24 ( \text{ENTER} ) ( \text{R/S} )</td>
<td>4.20</td>
</tr>
<tr>
<td>16 ( \text{ENTER} ) ( \text{R/S} )</td>
<td>5.00 ( x^2 )</td>
</tr>
</tbody>
</table>

**Normal Distribution**

The normal (or Gaussian) distribution is an important tool in statistics and business analysis. The following HP-12C program gives an approximation to the upper tail area \( Q \) under a standardized normal distribution curve, given \( x \). The upper tail area signifies the probability of occurrence of all values \( \geq x \).
Relative error less than 0.042% over the range 0 < x < 5.5

Reference:

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td></td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>00-</td>
</tr>
<tr>
<td>STO 0</td>
<td>01-440</td>
</tr>
<tr>
<td>8</td>
<td>02-8</td>
</tr>
<tr>
<td>3</td>
<td>03-3</td>
</tr>
<tr>
<td>X</td>
<td>04 - 20</td>
</tr>
<tr>
<td>---</td>
<td>--------</td>
</tr>
<tr>
<td>3</td>
<td>05 - 3</td>
</tr>
</tbody>
</table>
1. Key in program.
2. Key in $x$ and press $\text{R/S}$ to computed $Q(x)$. 

### REGISTERS

<table>
<thead>
<tr>
<th>n: Unused</th>
<th>i: Unused</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV: Unused</td>
<td>PMT: Unused</td>
</tr>
<tr>
<td>FV: Unused</td>
<td>$R_0$: $x$</td>
</tr>
<tr>
<td>$R_1$-$R_6$: Unused</td>
<td></td>
</tr>
</tbody>
</table>
3. Repeat step 2 for each new case.

**Example:** Find \( Q(x) \) for \( x = 1.18 \) and \( x = 2.1 \).

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.18 R/S</td>
<td>0.12 Q(1.18)</td>
</tr>
<tr>
<td>2.1 R/S</td>
<td>0.02 Q(2.1)</td>
</tr>
</tbody>
</table>

### Covariance

Covariance is a measure of the interdependence between paired variables \((x \text{ and } y)\). Like standard deviation, covariance may be defined for either a sample \((S_{xy})\) or a population \((S'_{xy})\) as follows:

\[
S_{xy} = r * s_x * s_y \\
S'_{xy} = r * s'_x * s'_y
\]

The following procedure finds the covariance of a sample \((S_{xy})\) and of a population \((S'_{xy})\):

1. Press \( \text{f} \) CLEAR \( \text{REG} \).
2. Key in the \( y \)-values and press \( \text{ENTER} \).
3. Key in the \( x \)-values and press \( \Sigma + \). Repeat steps 2 and 3 for all data pairs.
4. Press \( g \) \( \Sigma \) \( \text{x} \) \( \text{ENTER} \) \( g \) \( \hat{y} \) \( \text{R} \) \( \text{R} \) \( \text{x} \) to obtain the value of \( S_{xy} \).
5. Press \( \text{RCL} \) \( 1 \) \( \text{1} \) \( \text{1} \) \( \text{RCL} \) \( \pm \) \( \text{x} \) to obtain \( S'_{xy} \).

**Example 1:** Find the sample covariance \((S_{xy})\) and population covariance \((S'_{xy})\) for the following paired variables:

| \( x_i \) | 26 | 30 | 44 | 50 | 62 | 68 | 74 |
|\( y_i \) | 92 | 85 | 78 | 81 | 54 | 51 | 40 |

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{f} ) CLEAR ( \text{REG} ) ( 92 ) ( \text{ENTER} ) ( \Sigma + ) ( 85 ) ( \text{ENTER} ) ( \Sigma + ) ( 78 ) ( \text{ENTER} ) ( \Sigma + ) ( 81 ) ( \text{ENTER} ) ( \Sigma + )</td>
<td>7.00 Total number of entries.</td>
</tr>
</tbody>
</table>
Try the previous example using the following HP-12C program:

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>f P/R</code></td>
<td></td>
</tr>
<tr>
<td><code>f CLEAR PRGM</code></td>
<td>00-</td>
</tr>
<tr>
<td><code>Σ+</code></td>
<td>01- 49</td>
</tr>
<tr>
<td><code>g GTO</code> 00</td>
<td>02-43, 33 00</td>
</tr>
<tr>
<td><code>g s</code></td>
<td>03- 43 48</td>
</tr>
<tr>
<td><code>x</code></td>
<td>04- 20</td>
</tr>
<tr>
<td><code>ENTER</code></td>
<td>05- 36</td>
</tr>
<tr>
<td><code>g ´r</code></td>
<td>06- 43 2</td>
</tr>
<tr>
<td><code>R↓</code></td>
<td>07- 33</td>
</tr>
<tr>
<td><code>x</code></td>
<td>08- 20</td>
</tr>
<tr>
<td><code>R/S</code></td>
<td>09- 31</td>
</tr>
<tr>
<td><code>RCL</code> 1</td>
<td>10- 45 1</td>
</tr>
<tr>
<td><code>t</code></td>
<td>11- 1</td>
</tr>
<tr>
<td><code>-</code></td>
<td>12- 30</td>
</tr>
<tr>
<td><code>RCL</code></td>
<td>13- 45 1</td>
</tr>
<tr>
<td><code>+</code></td>
<td>14- 10</td>
</tr>
<tr>
<td><code>-</code></td>
<td>15- 20</td>
</tr>
<tr>
<td><code>g GTO</code> 00</td>
<td>16-43, 33 00</td>
</tr>
<tr>
<td><code>f P/R</code></td>
<td></td>
</tr>
</tbody>
</table>

REGISTERS
1. Key in the program.
2. Press CLEAR. 
3. Key in the y-value and press ENTER. 
4. Key in the x-value and press R/S. Repeat steps 3 and 4 for all data pairs. 
5. Press g GTO 03 R/S to obtain the value of $S_{xy}$. 
6. Press R/S to obtain $S'_{xy}$. 
7. For a new case, go to step 2. 

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f CLEAR REG</td>
<td></td>
</tr>
<tr>
<td>ENTER 26 R/S</td>
<td>7.00 Total number of entries.</td>
</tr>
<tr>
<td>ENTER 30 R/S</td>
<td></td>
</tr>
<tr>
<td>ENTER 44 R/S</td>
<td></td>
</tr>
<tr>
<td>ENTER 50 R/S</td>
<td></td>
</tr>
<tr>
<td>ENTER 62 R/S</td>
<td></td>
</tr>
<tr>
<td>ENTER 68 R/S</td>
<td></td>
</tr>
<tr>
<td>ENTER 74 R/S</td>
<td></td>
</tr>
<tr>
<td>g GTO 03 R/S</td>
<td>-354.14 $S_{xy}$</td>
</tr>
<tr>
<td>R/S</td>
<td>-303.55 $S'_{xy}$</td>
</tr>
</tbody>
</table>

**Permutation**

A permutation is an ordered subset of a set of distinct objects. The number of possible permutations, each containing $n$ objects, that can be formed from a collection of $m$ distinct objects is given by:
\[ mP_n = \frac{m!}{(m-n)!} \]

where \( m, n \) are integers and \( 69 \geq m \geq n \geq 0 \).

Use the following HP-12C program to calculate the number of possible permutations.

<table>
<thead>
<tr>
<th>KEYS</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>```</td>
<td>```</td>
</tr>
</tbody>
</table>

1. Key in the program.
2. Key in \( m \) and press \( \text{ENTER} \).
3. Key in \( n \) and press \( \text{R/S} \) to calculate \( mP_n \).
4. For a new case go to step 2.

**Example:** How many ways can 10 people be seated on a bench if only 4 seats are available?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
</table>

120
Combination

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing \( n \) objects, that can be formed from a collection of \( m \) distinct objects is given by:

\[
mCn = \frac{m!}{(m-n)!n!}
\]

Where \( m, n \) are integers and \( 69 \geq m \geq n \geq 0 \).

Use the following HP-12C to calculate the number of possible combinations.

<table>
<thead>
<tr>
<th>KEYS</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{f} )</td>
<td>( \text{P/R} )</td>
</tr>
<tr>
<td>( \text{f} )</td>
<td>( \text{CLEAR PRGM} )</td>
</tr>
<tr>
<td>STO</td>
<td>0</td>
</tr>
<tr>
<td>( x \times y )</td>
<td></td>
</tr>
<tr>
<td>( g )</td>
<td>( \text{nl} )</td>
</tr>
<tr>
<td>( g )</td>
<td>( \text{LSTX} )</td>
</tr>
<tr>
<td>RCL</td>
<td>0</td>
</tr>
<tr>
<td>( - )</td>
<td></td>
</tr>
<tr>
<td>( g )</td>
<td>( \text{nl} )</td>
</tr>
<tr>
<td>RCL</td>
<td>0</td>
</tr>
<tr>
<td>( g )</td>
<td>( \text{nl} )</td>
</tr>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>( + )</td>
<td></td>
</tr>
<tr>
<td>( g )</td>
<td>( \text{GTO 00} )</td>
</tr>
<tr>
<td>( \text{f} )</td>
<td>( \text{P/R} )</td>
</tr>
</tbody>
</table>

REGISTERS

- \( n \) Unused
- \( I \) Unused
- PV Unused
- PMT Unused
1. Key in the program.
2. Key in \( m \) and press \( \text{ENTER} \).
3. Key in \( n \) and press \( \text{R/S} \) to calculate \( mC_n \).
4. For a new case, go to step 2.

**Example:** A manager wants to choose a committee of three people from the seven engineers working for him. In how many different ways can the committee be selected?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 ENTER</td>
<td>35.00</td>
</tr>
<tr>
<td>3 R/S</td>
<td>( \binom{7}{3} )</td>
</tr>
</tbody>
</table>

**Random Number Generator**

This HP-12C program calculates uniformly distributed pseudo-random numbers \( u_i \) in the range

\[ 0 < u_i < 1. \]

The following method is used:

- \( u_i + 1 = \text{fractional part of } (997 u_i) \)
- where \( i = 0, 1, 2, \ldots \)
- \( u_0 = 0.5284163 \times \text{(seed)} \), "Other seeds may be selected but the quotient of \( (\text{seed} \times 10^7) \) divided by two or five must not be an integer. Also, it would be wise to statistically test other seeds before using them."

The period of this generator has a length of 500,000 numbers and the generator passes the frequency test (chi Square) for uniformity, the serial test and the run test. The most significant digits (the left hand digits) are the most random digits. The right most digits are significantly less random.

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f [P/R]</td>
<td></td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>00-</td>
</tr>
<tr>
<td>( \times )</td>
<td>01- 48</td>
</tr>
<tr>
<td>5</td>
<td>02- 5</td>
</tr>
</tbody>
</table>
1. Key in the program.
2. To generate a random number, press \( \text{R/S} \).
3. Repeat step 2 as many times as desired.

**Example:** Generate a sequence of 5 random numbers.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{R/S} )</td>
<td>0.83</td>
</tr>
<tr>
<td>( \text{R/S} )</td>
<td>0.83</td>
</tr>
<tr>
<td>( \text{R/S} )</td>
<td>0.83</td>
</tr>
<tr>
<td>( \text{R/S} )</td>
<td>0.83</td>
</tr>
<tr>
<td>( \text{R/S} )</td>
<td>0.83</td>
</tr>
</tbody>
</table>
Personal Finance

Homeowners Monthly Payment Estimator

It is often useful, when comparison shopping for a mortgage or determining the appropriate price range of houses to consider, to be able to quickly estimate the monthly payment given the purchase price, tax rate per $1000, percent down, interest rate and term of the loan.

The calculation assumes that the assessed value is 100% of the sales price and does not take into account financing of the closing costs.

A simple keystroke procedure may be used to calculate the monthly payment:

1. Press $\boxed{g}$ \boxed{END} and press \boxed{f} \boxed{CLEAR} \boxed{FIN}.
2. Key in the annual interest rate and press $\boxed{g}$ \boxed{12+}.
3. Key in the term of the loan (in years) and press $\boxed{g}$ \boxed{12x}.
4. Key in the purchase prices and press \boxed{STO} \boxed{1}.
5. Key in the percent down and press $\boxed{\%}$ $\boxed{-}$ \boxed{PV}.
6. Key in the tax rate in dollars per thousand and press $\boxed{RCL}$ \boxed{1} $\boxed{x}$ 12000 $\boxed{+}$ \boxed{CHS} \boxed{ENTER} \boxed{PMT} \boxed{PMT} $\boxed{+}$. (A negative sign is the convention for cash paid out).

Example: What would your monthly payments be on a $65,000 house in a neighborhood with a $25 per thousand tax rate and a 10 3/4 % interest rate on a 35 year loan with 10% down?

<table>
<thead>
<tr>
<th>Keystrokes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\boxed{g}$ \boxed{END}</td>
</tr>
<tr>
<td>\boxed{f} \boxed{CLEAR} \boxed{FIN}</td>
</tr>
<tr>
<td>10.75 $\boxed{g}$ \boxed{12+}</td>
</tr>
<tr>
<td>35 $\boxed{g}$ \boxed{12x}</td>
</tr>
<tr>
<td>65000 \boxed{STO} \boxed{1}</td>
</tr>
<tr>
<td>10 $\boxed{%}$ $\boxed{-}$ \boxed{PV}</td>
</tr>
<tr>
<td>25 \boxed{RCL} \boxed{1} $\boxed{x}$</td>
</tr>
<tr>
<td>12000 $\boxed{+}$ \boxed{CHS}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90 Monthly interest rate.</td>
</tr>
<tr>
<td>420.00 Months of loan.</td>
</tr>
<tr>
<td>65,000.00 Purchase price.</td>
</tr>
<tr>
<td>58,800.00 Mortgage balance.</td>
</tr>
<tr>
<td>-135.42 Approximate monthly taxes.</td>
</tr>
</tbody>
</table>
The following HP-12C program may be used instead of the above.

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
<td>00-</td>
</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>01- 43 8</td>
</tr>
<tr>
<td>g END</td>
<td>02- 45 1</td>
</tr>
<tr>
<td>RCL 1</td>
<td>03- 45 2</td>
</tr>
<tr>
<td>RCL 2</td>
<td>04- 25</td>
</tr>
<tr>
<td>-</td>
<td>05- 30</td>
</tr>
<tr>
<td>PV</td>
<td>06- 13</td>
</tr>
<tr>
<td>ENTER</td>
<td>07- 36</td>
</tr>
<tr>
<td>g LSTX</td>
<td>08- 43 36</td>
</tr>
<tr>
<td>+</td>
<td>09- 40</td>
</tr>
<tr>
<td>RCL 3</td>
<td>10- 45 3</td>
</tr>
<tr>
<td>x</td>
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</tr>
<tr>
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<td>3</td>
<td>15- 3</td>
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<tr>
<td>+</td>
<td>16- 10</td>
</tr>
<tr>
<td>CHS</td>
<td>17- 16</td>
</tr>
<tr>
<td>ENTER</td>
<td>18- 36</td>
</tr>
<tr>
<td>PMT</td>
<td>19- 14</td>
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<tr>
<td>PMT</td>
<td>20- 14</td>
</tr>
<tr>
<td>+</td>
<td>21- 40</td>
</tr>
<tr>
<td>g GTO 00</td>
<td>22-43, 33 00</td>
</tr>
<tr>
<td>f P/R</td>
<td></td>
</tr>
</tbody>
</table>

125
1. Key in the program.

2. Press $\text{f CLEAR FIN}$.

3. Key in the annual interest rate and press $\text{g 12+}$.

4. Key in the term of the loan in years and press $\text{g 12x}$.

5. Key in the purchase price and press $\text{STO 1}$.

6. Key in the percent down and press $\text{STO 2}$.

7. Key in the tax rate in dollars per thousand and press $\text{STO 3}$.

8. To calculate the approximate monthly payment, press $\text{R/S}$.

9. For a new case, store only the new variables by performing steps 3 thru 7 as needed. Press $\text{R/S}$ for the new approximate monthly payment.

**Example:** Solve the previous example using the HP-12C program.

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{f CLEAR FIN}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.75 $\text{g 12+}$</td>
<td>0.90</td>
<td>Monthly interest.</td>
</tr>
<tr>
<td>35 $\text{g 12x}$</td>
<td>420.00</td>
<td>Months of loan.</td>
</tr>
<tr>
<td>65000 STO 1</td>
<td>65,000.00</td>
<td>Purchase price.</td>
</tr>
<tr>
<td>10 STO 2</td>
<td>10.00</td>
<td>Percent down.</td>
</tr>
<tr>
<td>25 STO 3</td>
<td>25.00</td>
<td>Tax rate per thousand.</td>
</tr>
<tr>
<td>\text{R/S}</td>
<td>-672.16</td>
<td>Approximate monthly payment.</td>
</tr>
</tbody>
</table>

What would the approximate payment be if the loan was at 10% interest?

10 $\text{g 12+ R/S}$ -638.33 Approximate monthly payment.

What if the down payment is increased to 20%?

20 STO 2 R/S -582.45 Approximate monthly payment.
Tax-Free Individual Retirement (IRA) of Keogh Plan.

The advent of tax-free retirement accounts (IRA or Keogh) has resulted in considerable benefits for many persons who are not able to participate in group profit sharing or retirement plans. The savings due to tax-free status are often considerable, but complex to calculate. Required data are: the years to retirement, the total annual investment, the compound annual interest rate of the investment, and an assumed tax rate which would be paid on a similar non-tax free investment. This program calculates:

1. The future cash value of the tax-free investment.
2. The total cash paid in.
3. The total dividends paid.
4. The future value of the investment at retirement, assuming that after retirement you withdrew the money at a rate which causes the money to be taxed at 1/2 the rate at which it would otherwise have been taxed during the pay in period.
5. The diminished purchasing power assuming a given annual inflation rate.
6. The future value of a comparable taxable investment.
7. The diminished purchasing power of a comparable taxable investment.

Notes:

- The calculations run from the beginning of the first year to the end of the last year.
- The interest (annual yield), i, should be entered to as many significant figures as possible for maximum accuracy.
- The assumed 10% annual inflation rate may be changed by modifying the program at lines 19 and 20.
- The assumed tax rate used to calculate the after tax value of the tax-free investment may be changed by modifying the program at line 9.

<table>
<thead>
<tr>
<th>KEYSTROKES</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f [P/R]</td>
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</tr>
<tr>
<td>f CLEAR PRGM</td>
<td>00-</td>
</tr>
<tr>
<td>RCL n</td>
<td>01- 45 11</td>
</tr>
<tr>
<td>RCL PMT</td>
<td>02- 45 14</td>
</tr>
<tr>
<td>x</td>
<td>03- 20</td>
</tr>
<tr>
<td>R/S</td>
<td>04- 31</td>
</tr>
<tr>
<td>+</td>
<td>05- 40</td>
</tr>
<tr>
<td>R/S</td>
<td>06 - 31</td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
</tr>
<tr>
<td>RCL 1</td>
<td>07 - 45 1</td>
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<tr>
<td>▼</td>
<td>08 - 48</td>
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<td>5</td>
<td>09 - 5</td>
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<tr>
<td>%</td>
<td>10 - 25</td>
</tr>
<tr>
<td>CHS</td>
<td>11 - 16</td>
</tr>
<tr>
<td>1</td>
<td>12 - 1</td>
</tr>
<tr>
<td>+</td>
<td>13 - 40</td>
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<td>X</td>
<td>15 - 20</td>
</tr>
<tr>
<td>R/S</td>
<td>16 - 31</td>
</tr>
<tr>
<td>▼</td>
<td>17 - 1</td>
</tr>
<tr>
<td>1</td>
<td>18 - 48</td>
</tr>
<tr>
<td>0</td>
<td>19 - 1</td>
</tr>
<tr>
<td>▼</td>
<td>20 - 0</td>
</tr>
<tr>
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<td>21 - 45 11</td>
</tr>
<tr>
<td>y^X</td>
<td>22 - 21</td>
</tr>
<tr>
<td>+</td>
<td>23 - 10</td>
</tr>
<tr>
<td>R/S</td>
<td>24 - 31</td>
</tr>
<tr>
<td>RCL i</td>
<td>25 - 45 12</td>
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<tr>
<td>1</td>
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<td>RCL 1</td>
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</tr>
<tr>
<td>%</td>
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</tr>
<tr>
<td>√</td>
<td>29 - 30</td>
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<tr>
<td>X</td>
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<tr>
<td>1</td>
<td>31 - 12</td>
</tr>
<tr>
<td>FV</td>
<td>32 - 15</td>
</tr>
<tr>
<td>R/S</td>
<td>33 - 31</td>
</tr>
<tr>
<td>g GTO</td>
<td>34 - 43, 33 17</td>
</tr>
<tr>
<td>f P/R</td>
<td></td>
</tr>
</tbody>
</table>
1. Key in the program.

2. Press $\text{f CLEAR REG}$ and press $\text{g BEG}$.

3. Key in the tax rate as a percentage and press $\text{STO}$.  

4. Key in years to retirement and press $\text{n}$.

5. Key in the interest rates as a percentage and press $\text{i}$.

6. Key in the annual payment and press $\text{CHS PMT}$.

7. Press $\text{FV}$ to calculate the future value of the tax free investment.

8. Press $\text{R/S}$ to compute the total cash paid in.

9. Press $\text{R/S}$ to compute the total dividends paid.

10. Press $\text{R/S}$ to compute the future value when, after retirement, money is withdrawn at a rate causing the tax rate to equal 1/2 the rate paid during the pay-in period.

11. Press $\text{R/S}$ to compute the diminished purchasing power, in terms of today's dollars, of the future value assuming a 10% annual inflation rate.

12. Press $\text{R/S}$ to compute the future value of an ordinary tax investment.

13. Press $\text{R/S}$ to compute the diminished purchasing power of the ordinary tax investment.

**Example:** Assuming a 35 year investment period with a dividend rate of 8.175% and a tax rate of 40%.

1. If you invest $1500 each year in a tax free account, what will its value be at retirement?

2. How much cash will be paid in?

3. What will be the value of the earned dividends?

4. After retirement, if you withdrew cash form the account at a rate such that it will be taxed at a rate equal to one-half the rate paid during the pay-in period, what will be the after-tax value?

5. What is the diminished purchasing power of that amount, in today's dollars, assuming 10% annual inflation?
6. If you invest the same amount ($1500, *after taxes for a not-Keogh or IRA account.*) each year with dividends taxed as ordinary income, what will be the total tax-paid cash at retirement?

7. What is the purchasing power of that figure in terms of today's dollars?

<table>
<thead>
<tr>
<th>Key strokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.CLR REG</td>
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<tr>
<td>35 4 YA</td>
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<td>8.175 1</td>
<td>8.18</td>
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<td>1500 CHS PMT</td>
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<td>8,276.30</td>
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<td>R/S</td>
<td>139,360.09</td>
</tr>
<tr>
<td>R/S</td>
<td>4,959.00</td>
</tr>
</tbody>
</table>

**Stock Portfolio Evaluation and Analysis**

This program evaluates a portfolio of stocks given the current market price per share and the annual dividend. The user inputs the initial purchase price of a stock, the number of shares, the beta coefficient*, the annual dividend, and the current market price for a portfolio of any size. The program returns the percent change in value of each stock and the valuation and beta coefficient* of the entire portfolio. Output includes the original portfolio value, the new portfolio value, the percent change in the value and the annual dividend and yield as a percent of the current market value. The overall beta coefficient of the portfolio is also calculated.

*The beta coefficient is a measure of a stock variability (risk) compared to the market in general. Beta values for individual stocks can be acquired from brokers, investment publications or the local business library.

**Notes:**
• Prices are input in the form $XXX.ND$ where $N$ is the numerator and $D$ is the Denominator of the fractional portion of the price, e.g. 25 5/8 is input as 25.58.
• The beta coefficient analysis is optional. Key in 1.00 if beta is not to be analyzed.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>f P/R</td>
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</tr>
<tr>
<td>f CLEAR PRGM</td>
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<tr>
<td>STO 6</td>
<td>01- 44 6</td>
</tr>
<tr>
<td>g FRAC</td>
<td>02- 43 24</td>
</tr>
<tr>
<td>g x=0</td>
<td>03- 43 35</td>
</tr>
<tr>
<td>g GTO 15</td>
<td>04-43, 33 15</td>
</tr>
<tr>
<td>fi</td>
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<td>0</td>
<td>06- 0</td>
</tr>
<tr>
<td>x</td>
<td>07- 20</td>
</tr>
<tr>
<td>g INTG</td>
<td>08- 43 25</td>
</tr>
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<td>g LSTX</td>
<td>09- 43 36</td>
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<td>g FRAC</td>
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<td>t</td>
<td>12- 1</td>
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<td>13- 0</td>
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<tr>
<td>÷</td>
<td>14- 10</td>
</tr>
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<td>15- 45 6</td>
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<tr>
<td>g INTG</td>
<td>16- 43 25</td>
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<td>+</td>
<td>17- 40</td>
</tr>
<tr>
<td>RCL 4</td>
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<tr>
<td>g x=0</td>
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<tr>
<td>g GTO 38</td>
<td>20-43, 33 38</td>
</tr>
<tr>
<td>STO - 4</td>
<td>21-44 30 4</td>
</tr>
<tr>
<td>x</td>
<td>22- 20</td>
</tr>
<tr>
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<td>23- 45 7</td>
</tr>
<tr>
<td>x</td>
<td>24- 20</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td>x</td>
<td>y</td>
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<tr>
<td>RCL 7</td>
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<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td>STO + 0</td>
<td>25-44</td>
</tr>
<tr>
<td>STO + 1</td>
<td>29-44</td>
</tr>
<tr>
<td>STO + 3</td>
<td>32-44</td>
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<tr>
<td>RCL 5</td>
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<tr>
<td>g LSTX</td>
<td>34-43</td>
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<tr>
<td>Δ%</td>
<td>35-24</td>
</tr>
<tr>
<td>R/S</td>
<td>36-31</td>
</tr>
<tr>
<td>g GTO 01</td>
<td>37-43, 33</td>
</tr>
<tr>
<td>+</td>
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<tr>
<td>STO 5</td>
<td>42-44</td>
</tr>
<tr>
<td>STO + 2</td>
<td>43-44</td>
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<tr>
<td>STO 4</td>
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<tr>
<td>R/S</td>
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<td>RCL 2</td>
<td>48-45</td>
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<tr>
<td>R/S</td>
<td>49-31</td>
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<td>RCL 0</td>
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<tr>
<td>R/S</td>
<td>51-31</td>
</tr>
<tr>
<td>Δ%</td>
<td>52-24</td>
</tr>
</tbody>
</table>
Instructions:
1. Key in the program.
2. Initialize the program by pressing $\text{f CLEAR REG}$.
3. Key in the number of shares of a stock and press $\text{ENTER}$.
4. Key in the initial purchase of the stock and press $\text{R/S}$.
5. Key in the beta coefficient of the stock and press $\text{ENTER}$.
6. Key in the annual dividend of the stock and press $\text{ENTER}$.
7. Key in the present price of the stock and press $\text{R/S}$. The display will show the percent change in the stock value.
8. Repeat steps 3 through 7 until all the stocks are entered.
9. Next, to evaluate the entire portfolio, press \( g \) \( \text{GTO} \) 48.

10. Press \( \text{R/S} \) to see the initial portfolio value.

11. Press \( \text{R/S} \) to see the present portfolio value.

12. Press \( \text{R/S} \) to see the percent change in value.

13. Press \( \text{R/S} \) to see the total yearly dividend.

14. Press \( \text{R/S} \) to see the annual dividend yield as a percent of the current market value.

15. Press \( \text{R/S} \) to see the beta coefficient of the portfolio.

16. For a new case return to step 2.

**Example:** Evaluate the following portfolio:

<table>
<thead>
<tr>
<th>Number of Shares Held</th>
<th>Initial Purchase Price</th>
<th>Beta Coefficient</th>
<th>Annual Dividend</th>
<th>Present Market Price</th>
<th>Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>25 5/8</td>
<td>.8</td>
<td>$1.70</td>
<td>27 1/4</td>
<td>Int’l Heartburn</td>
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<tr>
<td>200</td>
<td>30 1/4</td>
<td>1.2</td>
<td>$2.10</td>
<td>33 1/2</td>
<td>P. D. Q.</td>
</tr>
<tr>
<td>50</td>
<td>89 7/8</td>
<td>1.3</td>
<td>$4.55</td>
<td>96 1/8</td>
<td>Datacrunch</td>
</tr>
<tr>
<td>500</td>
<td>65 1/4</td>
<td>.6</td>
<td>$3.50</td>
<td>64 3/8</td>
<td>N.W. Sundial</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 ) ( \text{CLEAR} ) ( \text{REG} )</td>
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<td></td>
</tr>
<tr>
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<td>ENTER</td>
<td>100.00</td>
</tr>
<tr>
<td>25.58</td>
<td>R/S</td>
<td>1.00</td>
</tr>
<tr>
<td>.8</td>
<td>ENTER</td>
<td>0.80</td>
</tr>
<tr>
<td>1.70</td>
<td>ENTER</td>
<td>1.70</td>
</tr>
<tr>
<td>27.14</td>
<td>R/S</td>
<td>6.34</td>
</tr>
<tr>
<td>200</td>
<td>ENTER</td>
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</tr>
<tr>
<td>30.14</td>
<td>R/S</td>
<td>1.00</td>
</tr>
<tr>
<td>1.2</td>
<td>ENTER</td>
<td>1.20</td>
</tr>
<tr>
<td>2.10</td>
<td>ENTER</td>
<td>2.10</td>
</tr>
<tr>
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<tr>
<td>Stock Name</td>
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<tr>
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<td>-----</td>
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</tr>
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<td>89.78</td>
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<td>1.30</td>
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<tr>
<td>96.18</td>
<td>6.95</td>
<td>500.00</td>
</tr>
<tr>
<td>64.38</td>
<td>-1.34</td>
<td>3.50</td>
</tr>
</tbody>
</table>

**Calculations**

- **Original value**: 45,731.25
- **Present value**: 46,418.75
- **Total yearly dividend**: 2,567.50
- **Annual dividend yield**: 5.53
- **Portfolio beta coefficient**: 0.77
Canadian Mortgages

In Canada, interest is compounded semi-annually with payments made monthly. This results in a different monthly mortgage factor than is used in the United States and preprogrammed into the HP-12C. This difference can be easily handled by the addition of a few keystrokes. For any problem requiring an input for \( i \), the Canadian mortgage factor is calculated first and then this value is entered in for \( i \) in the calculation to give the answer for Canada. The keystrokes to calculate the Canadian Mortgage factor are:

1. Press \( \boxed{\text{f CLEAR FIN} \ g \ END} \).
2. Key in 6 and press \( \boxed{n} \).
3. Key in 200 and press \( \boxed{\text{ENTER PV}} \).
4. Key in the annual interest rate as a percentage and press \( \boxed{+ \ CHS \ FV} \).
5. Press \( \boxed{\text{i}} \).

The Canadian mortgage factor is now stored in \( \boxed{i} \) for future use. The examples below show how this factor is used for \( \boxed{i} \) in Canadian mortgage problems.

**Periodic Payment Amount**

**Example 1:** What is the monthly payment required to fully amortize a 30-year, $30,000 Canadian mortgage if the interest rate is 9%?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
<th>Description</th>
</tr>
</thead>
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<td>( \boxed{\text{f CLEAR FIN}} )</td>
<td>( 0.74 )</td>
<td>Canadian mortgage factor</td>
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<td>( \boxed{\text{g END}} )</td>
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<td></td>
</tr>
<tr>
<td>( \boxed{6 \ n \ 200 \ \text{ENTER PV}} )</td>
<td>( 360.00 )</td>
<td>Total monthly periods in mortgage life</td>
</tr>
<tr>
<td>( \boxed{9 \ + \ CHS \ FV} )</td>
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<td></td>
</tr>
<tr>
<td>( \boxed{30 \ g \ 12x} )</td>
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<td></td>
</tr>
<tr>
<td>( \boxed{30000 \ FV \ 0 \ FV} )</td>
<td>( -237.85 )</td>
<td>Monthly payment</td>
</tr>
<tr>
<td>( \boxed{\text{PMT}} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Number of Periodic Payments to Fully Amortize a Mortgage

Example 2: An investor can afford to pay $440 per month on a $56,000 Canadian Mortgage. If the annual interest rate is 9 1/4 %, how long will it take to completely amortize this mortgage?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEAR FIN</td>
<td>0.76</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>6 n 200 ENTER PV</td>
<td>-440.00</td>
</tr>
<tr>
<td>9.25 CHS FV i</td>
<td></td>
</tr>
<tr>
<td>440 CHS PMT</td>
<td>437.00</td>
</tr>
<tr>
<td>56000 FV 0 FV n</td>
<td></td>
</tr>
</tbody>
</table>

Effective Interest Rate (Yield)

Example 3: A Canadian mortgage has monthly payments of $612.77 with a maturity of 25 years. The principal amount is $75,500. What is the annual interest rate?

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>CLEAR FIN</td>
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<tr>
<td>25 g 12 x</td>
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</tr>
<tr>
<td>612.77 CHS PMT</td>
<td></td>
</tr>
<tr>
<td>75500 PV i</td>
<td></td>
</tr>
<tr>
<td>6 n 0 PMT</td>
<td>8.75</td>
</tr>
<tr>
<td>200 CHS PV</td>
<td></td>
</tr>
<tr>
<td>FV RCL FV +</td>
<td></td>
</tr>
</tbody>
</table>

Balance Remaining at End of Specified Period

Example 4: A Canadian mortgage has monthly payments of $612.77 at 8.75% interest. The principal amount is $75,500. What will be the outstanding balance remaining at the end of 10 years?

<table>
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</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>g END</td>
<td></td>
</tr>
<tr>
<td>6 n 200 ENTER PV</td>
<td></td>
</tr>
<tr>
<td>8.75 CHS PV</td>
<td></td>
</tr>
<tr>
<td>FV RCL FV</td>
<td></td>
</tr>
<tr>
<td>CLEAR</td>
<td>FIN</td>
</tr>
<tr>
<td>6</td>
<td>n</td>
</tr>
<tr>
<td>8.75</td>
<td>+</td>
</tr>
<tr>
<td>612.77</td>
<td>CHS</td>
</tr>
<tr>
<td>10</td>
<td>g</td>
</tr>
<tr>
<td>75500</td>
<td>PV</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 61,877.18 |
| Outstanding balance remaining at the end of 10 years. |
Learning Curve for Manufacturing Costs

Many production process costs vary with output according to the "learning curve" equation. The production team becomes more proficient in manufacturing a given item as more and more of them are fabricated and costs may be expected to decrease by a predictable amount. The learning factor, $r$, characterizes the learning curve. For instance, if $r=.80$ the curve is called an 80% learning curve.

It is readily apparent that the learning, or experience curve, has many uses in setting production standards, forecasting costs, setting prices, etc. Note, however, that the learning factor may change, especially after large numbers have been produced.

If the cost of the first unit of a run, $C_1$, and the learning curve factor, $r$, are known, the following procedure can be used to calculate the cost of the $n$th item:

1. Key in the cost of the first item, $C_1$ and press $\text{ENTER}$.
2. Key in the number of units produced, $n$, and press $\text{ENTER}$.
3. Key in the learning factor, $r$, and press $\frac{g\ \ln}{2\ \ln} \div$.
4. Then press $\sqrt[y]{x}$ to calculate the cost of the $n$th unit, $C_n$.

**Example 1:** An electronic manufacturer begins a pilot run on a new instrument. From past experience he expects the process to have a learning factor, $r$, or 0.90. If the first unit costs $875 to produce, what is the expected cost of the 100th unit?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>875 $\text{ENTER}$</td>
<td>875.00</td>
</tr>
<tr>
<td>100 $\text{ENTER}$</td>
<td>100.00</td>
</tr>
<tr>
<td>.9 $\frac{g\ \ln}{2\ \ln} \div$</td>
<td>-0.15</td>
</tr>
<tr>
<td>$\sqrt[y]{x}$</td>
<td>434.51 Cost of the 100th unit.</td>
</tr>
</tbody>
</table>

If the cost of the first unit, $C_1$, and the $n$th unit, $C_n$, are known the learning factor may be calculated. In addition, it is possible to calculate $C_{ij}$, the average cost of the $i$th thru $j$th unit. These calculations may be rapidly done with the following HP-12C program:
<table>
<thead>
<tr>
<th>f</th>
<th>P/R</th>
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</thead>
<tbody>
<tr>
<td>f</td>
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<tr>
<td>g</td>
<td>LN 01- 43 23</td>
</tr>
<tr>
<td>2</td>
<td>02- 2</td>
</tr>
<tr>
<td>g</td>
<td>LN 03- 43 23</td>
</tr>
<tr>
<td>+</td>
<td>04- 10</td>
</tr>
<tr>
<td>STO 2</td>
<td>05- 44 2</td>
</tr>
<tr>
<td>R→</td>
<td>06- 33</td>
</tr>
<tr>
<td>x⁻→y</td>
<td>07- 34</td>
</tr>
<tr>
<td>STO 1</td>
<td>08- 44 1</td>
</tr>
<tr>
<td>+</td>
<td>09- 10</td>
</tr>
<tr>
<td>g</td>
<td>LN 10- 43 23</td>
</tr>
<tr>
<td>RCL 2</td>
<td>11- 45 2</td>
</tr>
<tr>
<td>+</td>
<td>12- 10</td>
</tr>
<tr>
<td>g</td>
<td>e^x 13- 43 22</td>
</tr>
<tr>
<td>STO 2</td>
<td>14- 44 2</td>
</tr>
<tr>
<td>g</td>
<td>GTO 00 15- 43, 33 00</td>
</tr>
<tr>
<td>RCL 2</td>
<td>16- 45 2</td>
</tr>
<tr>
<td>g</td>
<td>LN 17- 43 23</td>
</tr>
<tr>
<td>2</td>
<td>18- 2</td>
</tr>
<tr>
<td>g</td>
<td>LN 19- 43 23</td>
</tr>
<tr>
<td>+</td>
<td>20- 10</td>
</tr>
<tr>
<td>y^x</td>
<td>21- 21</td>
</tr>
<tr>
<td>RCL 1</td>
<td>22- 45 1</td>
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<tr>
<td>x</td>
<td>23- 20</td>
</tr>
<tr>
<td>g</td>
<td>GTO 00 24- 43, 33 00</td>
</tr>
<tr>
<td>STO 3</td>
<td>25- 44 3</td>
</tr>
<tr>
<td>x⁻→y</td>
<td>26- 34</td>
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</table>

140
<table>
<thead>
<tr>
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<tr>
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<td>4</td>
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<tr>
<td>RCL 2</td>
<td>28</td>
<td>45</td>
</tr>
<tr>
<td>g LN 2</td>
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<td>43</td>
</tr>
<tr>
<td>+</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>g LN 1</td>
<td>31</td>
<td>43</td>
</tr>
<tr>
<td>+</td>
<td>32</td>
<td>10</td>
</tr>
<tr>
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<td>33</td>
<td>1</td>
</tr>
<tr>
<td>y^x</td>
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<td>40</td>
</tr>
<tr>
<td>RCL 3</td>
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<td>44</td>
</tr>
<tr>
<td>RCL 0</td>
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<td>21</td>
</tr>
<tr>
<td>RCL 0</td>
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<td>45</td>
</tr>
<tr>
<td>-</td>
<td>38</td>
<td>30</td>
</tr>
<tr>
<td>RCL 0</td>
<td>39</td>
<td>45</td>
</tr>
<tr>
<td>+</td>
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<td>10</td>
</tr>
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<td>45</td>
</tr>
<tr>
<td>RCL 3</td>
<td>42</td>
<td>45</td>
</tr>
<tr>
<td>-</td>
<td>43</td>
<td>30</td>
</tr>
<tr>
<td>+</td>
<td>44</td>
<td>10</td>
</tr>
<tr>
<td>RCL 1</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>x</td>
<td>46</td>
<td>20</td>
</tr>
<tr>
<td>GTO 00</td>
<td>47</td>
<td>0</td>
</tr>
<tr>
<td>P/R</td>
<td>48</td>
<td>0</td>
</tr>
<tr>
<td>GTO 00</td>
<td>49</td>
<td>43, 33</td>
</tr>
</tbody>
</table>

**REGISTERS**

- **n**: Unused
- **i**: Unused
- **PV**: Unused
- **PMT**: Unused
- **FV**: Unused
- **R0**: K+1
- **R1**: C1
- **R2**: r
- **R3**: /
- **R4**: /
1. Key in the program, (Note: If the average cost are not going to be calculated, lines 25 through 48 need not be keyed in).

2. To calculate \( r \), the learning factor, if \( C_1 \) and \( C_n \) are known:
   a. Key in \( C_1 \), the cost of the first unit and press \( \text{ENTER} \).
   b. Key in \( C_n \), the cost of the \( n \)th unit and press \( \text{ENTER} \).
   c. Key in \( n \), the number of units and press \( \text{R/S} \) to calculate \( r \) the learning factor.

3. To calculate the cost of the \( n \)th unit when \( C_1 \) and \( r \) are known:
   a. Key in \( C_1 \) and press \( \text{STO} \) 1. Key in \( r \) and press \( \text{STO} \) 2. (Note: This step may be skipped if step 2 has just been done).
   b. Key in the number of units, \( n \) and calculate \( C_n \), the cost of the \( n \)th unit by pressing \( \text{g} \), \( \text{GTO} \), 16 \( \text{R/S} \).

4. To calculate the average cost per unit of the \( i \)th through \( j \)th unit, \( C_{ij} \), if \( C_1 \) and \( r \) are known.
   a. Key in \( C_1 \) and press \( \text{STO} \) 1. Key in \( r \) and press \( \text{STO} \) 2. (Note: This step may be skipped if step 2 has just been done).
   b. Key in the number of the last unit of the batch, \( j \) and press \( \text{ENTER} \).
   c. Key in the number of the first unit of the batch, \( i \), and calculate the average cost per unit by pressing \( \text{g} \), \( \text{GTO} \), 25 \( \text{R/S} \).

Example 2: The electronic manufacturer cited in example 1 found that the 100th instrument actually cost $395 to manufacture. Find the actual learning factor, \( r \), the cost of the 500th unit and the average cost of units 500 thru 1000. (Recall that \( C_1 \) was $875).

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>875 ( \text{ENTER} )</td>
<td>875.00</td>
</tr>
<tr>
<td>395 ( \text{ENTER} )</td>
<td>395.00</td>
</tr>
<tr>
<td>100 ( \text{R/S} )</td>
<td>0.89 Actual ( r ).</td>
</tr>
<tr>
<td>500 ( \text{GTO} ), 16 ( \text{R/S} )</td>
<td>299.14 Cost of the 500th unit.</td>
</tr>
<tr>
<td>1000 ( \text{ENTER} )</td>
<td>1000.00</td>
</tr>
<tr>
<td>500 ( \text{GTO} ), 25 ( \text{R/S} )</td>
<td>280.00 Average cost of the 500th thru 1000th unit.</td>
</tr>
</tbody>
</table>
Queuing and Waiting Theory

Waiting lines, or queues, cause problems in many marketing situations. Customer goodwill, business efficiency, labor and space considerations are only some of the problems which may be minimized by proper application of queuing theory.

Although queuing theory can be complex and complicated subject, handheld calculators can be used to arrive at helpful decisions.

One common situation that we can analyze involves the case of several identical stations serving customers, where the customers arrive randomly in unlimited numbers. Suppose there are \( n \) (1 or more) identical stations serving the customers. \( \lambda \) is the arrival rate (Poisson input) and \( \mu \) is the service rate (exponential service). We will assume that all customers are served on a firstcome, first-served basis and wait in a single line (queue) then are directed to whichever station is available. We also will assume that no customers are lost from the queue. This situation, for instance, would be closely approximated by customers at some banking operations.

The formulas for calculating some of the necessary probabilities are too complex for simple keystroke solution. However, tables listing these probabilities are available and can be used to aid in quick solutions. Using the assumptions outlined above and a suitable table giving mean waiting time as a multiple of mean service (see page 512 of the Reference) the following keystroke solutions may be obtained:

1. Key in the arrival rate of customers, \( \lambda \), and press \( \text{ENTER} \).
2. Key in the service rate, \( \mu \), and press \( \div \) to calculate \( \rho \), the intensity factor. (Note \( \rho \) must be less than \( n \) for valid results, otherwise the queue will lengthen without limit).
3. Key in \( n \), the number of servers and press \( \div \) to calculate \( \rho/n \).
4. For a given \( n \) and \( \rho/n \) find the mean waiting time as a multiple of mean service time from the table. Key it in and press \( \text{ENTER} \).
5. Calculate the average waiting time in the queue by keying in the service rate, \( \mu \), and pressing \( \text{STO} \) 1 \( + \) \( \text{STO} \) 2.
6. Calculate the average waiting time in the system by pressing \( \text{RCL} \) 1 \( 1/x \) \( + \).
7. Key in \( \lambda \) and press \( \text{RCL} \) 2 \( x \) to calculate the average queue length.
8. Key in \( \rho \), then intensity factor (from step 2 above) and press \( \div \) to calculate the average number of customer in the system.

Reference:
Example 1: Bank customers arrive at a bank on an average of 1.2 customers per minute. They join a common queue for three tellers. Each teller completes a transaction at the rate of one customer every 2 minutes (0.5 customers per minute). What is the average waiting time in the queue? In the system? What is the average number of customers in the queue? In the system?

From Table 12.2, page 512 of the reference, the mean waiting time as a multiple of mean service time for \( n = 3 \), \( \rho/n = 0.8 \) is 1.079. (Note \( S \) is used instead of \( n \) in the reference’s notation).

If the number of servers is limited to one, with other conditions remaining the same (unlimited queue, Poisson arrival, exponential service), the average queue length can be readily calculated without reference to tables:

1. Key in the arrival rate, \( \lambda \), and press \( 1 \).
2. Key in the service rate, \( \mu \), and press \( 2 \) to calculate the average number of customers waiting in queue at any one time.
3. Press \( 1 \) to calculate the average waiting time.
4. Press \( 2 \) to calculate the average total time the customer spends in the system.
5. Press \( 1 \) to calculate the average number of customers in the system.

Example 2: A small grocery store has but a single check-out counter. Customers arrive at a rate of 1 every 2 minutes \( (\lambda = .5) \) and, on the average, customers can be checked out at a rate of .9 per minute \( (\mu) \).
What is the average number of customers in the waiting line at any time? The average waiting time? What is the average total time for a customer to wait and be checked out? The average number of customers in the system?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5 STO 1</td>
<td>0.50</td>
</tr>
<tr>
<td>9 STO 2 ÷ ENTER</td>
<td>0.56</td>
</tr>
<tr>
<td>ENTER 2 ( y^x ) ( x )</td>
<td>0.69 Average # customers waiting in queue.</td>
</tr>
<tr>
<td>RCL 1 +</td>
<td>1.39 Average waiting time.</td>
</tr>
<tr>
<td>RCL 2 1/x +</td>
<td>2.50 Average total time in the system.</td>
</tr>
<tr>
<td>RCL 1 x</td>
<td>1.25 Average # customers in system.</td>
</tr>
</tbody>
</table>

With an HP-12C program on can readily calculate the necessary probabilities for this type of problem (dispensing with the use of tables) and perform additional calculations as well.

<table>
<thead>
<tr>
<th>KEystrokes</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>f [P/R]</td>
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</tr>
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</tr>
<tr>
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<td>02-44 33 0</td>
</tr>
<tr>
<td>RCL - 0</td>
<td>03-45 48 0</td>
</tr>
<tr>
<td>RCL 0</td>
<td>04- 45 0</td>
</tr>
<tr>
<td>0</td>
<td>05- 0</td>
</tr>
<tr>
<td>g x( \leq y )</td>
<td>06- 43 34</td>
</tr>
<tr>
<td>g GTO 09</td>
<td>07-43, 33 09</td>
</tr>
<tr>
<td>g GTO 16</td>
<td>08-43, 33 16</td>
</tr>
<tr>
<td>+</td>
<td>09- 40</td>
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<tr>
<td>( y^x )</td>
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<tr>
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<td>Function</td>
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<td>g</td>
<td>GTO 01</td>
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<td>16-45 48 0</td>
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<tr>
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<td>17- 45 7</td>
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<td>19- 1</td>
</tr>
<tr>
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<td>20-45 48 0</td>
</tr>
<tr>
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<td>21- 45 7</td>
</tr>
<tr>
<td>+</td>
<td>22- 10</td>
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<td>-</td>
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</tr>
<tr>
<td>RCL 7</td>
<td>25- 45 7</td>
</tr>
<tr>
<td>g</td>
<td>n</td>
</tr>
<tr>
<td>+</td>
<td>27- 10</td>
</tr>
<tr>
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</tr>
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</tr>
<tr>
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<td>30- 40</td>
</tr>
<tr>
<td>1/x</td>
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<tr>
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</tr>
<tr>
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<td>35- 44 2</td>
</tr>
<tr>
<td>RCL · 0</td>
<td>36-45 48 0</td>
</tr>
<tr>
<td>$\times$</td>
<td>37- 20</td>
</tr>
<tr>
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<td>38- 45 7</td>
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<td>39-45 48 0</td>
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<td>40- 30</td>
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<td>x</td>
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<tr>
<td>-</td>
<td>58 - 30</td>
</tr>
<tr>
<td>x</td>
<td>59 - 20</td>
</tr>
<tr>
<td>g (e^x)</td>
<td>60 - 43 22</td>
</tr>
<tr>
<td>RCL 2</td>
<td>61 - 45 2</td>
</tr>
<tr>
<td>x</td>
<td>62 - 20</td>
</tr>
<tr>
<td>g (GTO) 53</td>
<td>63-43, 33 53</td>
</tr>
<tr>
<td>f P/R</td>
<td></td>
</tr>
</tbody>
</table>

REGISTERS

- **n**: Unused
- **I**: Unused
- **PV**: Unused
- **PMT**: Unused
- **FV**: Unused
- **R0**: \(K\)
- **R1**: \(P_0\)
- **R2**: \(P_b\)
- **R3**: \(T_q\)
- **R4**: \(L\)
- **R5**: \(T\)
- **R6**: Used, \(T_q\)
1. Key in the program and press \[ f \text{ CLEAR} \text{ REG} \] 0.
2. Key in the number of servers, \( n \) and press \[ \text{ STO} 7 \] 0.
3. Key in the arrival rate of customers, \( \lambda \), and press \[ \text{ STO} 8 \] 1.
4. Key in the service rate of each server, \( \mu \), and press \[ \text{ STO} 9 \] 2.
5. Press \[ \boxed{\div} \text{ STO} \boxed{\times} \] 0 to calculate and store \( \rho \), the intensity factor.
6. Press \[ \text{ R/S} \] to see \( T_q \), the average waiting time in the queue. Display \( P_0 \), probability that all servers are idle, by pressing \[ \text{ RCL} \[ 1 \] \]. Display \( P_b \), probability that all servers are busy by pressing \[ \text{ RCL} \[ 2 \] \].
7. Display \( L_q \), the average number waiting in the queue by pressing \[ \text{ RCL} \[ 3 \] \]. Display \( L \), the average number in the system (waiting and being served), by pressing \[ \text{ RCL} \[ 4 \] \]. Display \( T \), average total time through the system, by pressing \[ \text{ RCL} \[ 5 \] \]. \( T_q \), the average waiting time in the queue, may again be displayed by pressing \[ \text{ RCL} \[ 6 \] \].
8. If desired, calculate \( P(t) \), the probability of waiting longer than a given time, by keying in the time and pressing \[ \text{ R/S} \] 7.
9. Repeat step 7 for other times of interest.

**Example 3:** Using the data from example 1 of the keystroke solutions verify the data obtained. In addition, obtain \( P_0 \), the probability that none of the tellers are busy, and \( P_b \) the probability that all the tellers are busy.

What is the probability that a customer will have to wait 2 minutes or more?

<table>
<thead>
<tr>
<th>Keystrokes</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ f \text{ CLEAR} \text{ REG} ]</td>
<td>0.00</td>
</tr>
<tr>
<td>[ 3 \text{ STO} 0 \text{ STO} 7 ]</td>
<td>3.00 ( n )</td>
</tr>
<tr>
<td>[ 1.2 \text{ STO} 8 ]</td>
<td>1.20 ( \lambda )</td>
</tr>
<tr>
<td>[ 5 \text{ STO} 9 ]</td>
<td>0.50 ( \mu )</td>
</tr>
<tr>
<td>[ \boxed{\div} \text{ STO} \boxed{\times} \boxed{0} ]</td>
<td>2.40 ( \rho )</td>
</tr>
<tr>
<td>[ \text{ R/S} ]</td>
<td>2.16 ( T_q ) average waiting time in queue.</td>
</tr>
<tr>
<td>[ \text{ RCL} [ 1 ] ]</td>
<td>0.06 ( P_0 ) probability all servers are idle.</td>
</tr>
<tr>
<td>RCL</td>
<td>2</td>
</tr>
<tr>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>RCL</td>
<td>3</td>
</tr>
<tr>
<td>RCL</td>
<td>4</td>
</tr>
<tr>
<td>RCL</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>R/S</td>
</tr>
</tbody>
</table>
Appendix

Real Estate

Wrap-Around Mortgage

- \( n_1 \) = number of years remaining in original mortgage.
- \( PMT_1 \) = yearly payment of original mortgage.
- \( PV_1 \) = remaining balance of original mortgage.
- \( n_2 \) = number of years in wrap-around mortgage.
- \( PMT_2 \) = yearly payment of wrap-around mortgage.
- \( r \) = interest rate of wrap-around mortgage as a decimal.
- \( FV \) = balloon payment.

\[
P V_2 - P V_1 = \frac{PMT_2 [1 - (1 + r)^{-n_2}]}{r} - \frac{PMT_1 [1 - (1 + r)^{-n_1}]}{r} + FV(1 + r)^{-n_2}
\]

After-Tax Cash Flows

- \( ATCF_k \) = After-Tax Cash Flow for \( k \)th year.
- \( Int_k \) = interest for \( k \)th year.
- \( Dep_k \) = depreciation for \( k \)th year.
- \( r \) = appropriate tax rate.
- \( NOI \) = Net Operating Income.
- \( ATCF_k = NOI (1 - r) - 12 \times PMT + r \times (Int_k + Dep_k) \).

After-Tax Net Cash Proceeds of Resale

- \( CO \) = capital purchase.
- \( CPR \) = sales price - closing costs.
- \( r \) = marginal tax rate.
- \( NCPR \) = \( CPR \) - remaining balance of mortgage.
- \( ATNCPR = NCPR + r \times [(0.6 \text{ SL Dep.} - \text{Total Dep}) + 0.4 \times (CO - CPR)] \)
Lending

Loans with a constant amount paid towards Principal

- \( BAL_k \) = remaining balance after time period \( k \).
- \( CPMT \) = Constant payment to principal.
- \( BAL_k = PV - (k \times CPMT) \)
- Kth payment to interest = \( i \times (BAL_k) = (PMT)_k \)
- Kth total payment = \( CPMT + (PMT)_k \)

Add-On Interest Rate to APR

- \( r \) = add-on rate as a decimal.
- \( n \) = number of monthly payments.
- \( APR = 1200i \), where \( i \) is the solution in the following equation:

\[
\frac{n}{1 + \frac{n}{12} \times r} \times \frac{(1 + i)^n}{i} = \frac{1 - (1 + i)^{-n}}{i}
\]

Add-On to APR with Credit Life

- \( CL = \) credit life as decimal.
- \( AMT = \) loan amount.
- \( FC = \) finance charge.

\[
\left[ \frac{1 + \left( \frac{n}{12} \right) r}{1 - \left( \frac{n}{12} \right) CL \times \frac{n}{12} \times CL \times r} \right]
\]

\[
\frac{G \times n}{n} = PMT
\]

\[
\frac{G \times CL \times n}{12} = \text{amount of credit life}
\]
• \( FC = (G - AMT - CL) \)

Rule of 78’s Rebate
• \( PV = \) finance charge.
• \( I_k = \) interest charged at month \( k \).
• \( n = \) number of months in loan.
• 
\[
I_k = \frac{2(n - k + 1)}{n(n + 1)}PV
\]
• 
\[
Rebate = \frac{(n - k)I_k}{2}
\]
• \( BAL_k = (n - k) \times PMT - Rebate_k \)

Skipped Payments
• \( A = \) number of payments per year.
• \( B = \) number of years.
• \( C = \) annual percentage rate as decimal.
• \( D = \) periodic payment amount.
• \( E = \) loan amount.
• \( K = \) number of last payment before payments close the first time.
• \( L = \) number of skipped payment.

\[
D_{END} = \frac{E}{1 - \left(1 + \frac{C}{A}\right)^{AB}} \times \frac{\left[\left(1 + \frac{C}{A}\right)^{A} - 1\right]C}{\left[\left(1 + \frac{C}{A}\right)^{A} - \left(1 + \frac{C}{A}\right)^{K} + \left(1 + \frac{C}{A}\right)^{L - K}\right]}
\]

\[
D_{BEGIN} = \frac{D_{END}}{1 + \frac{C}{A}}
\]

Savings
Compounding Periods Different From Payment Periods

- \( C \) = number of compounding periods per year.
- \( P \) = number of payments periods per year.
- \( i \) = periodic interest rate, expressed as a percentage.
- \( r = \frac{i}{100} \), periodic interest rate expressed as a decimal.
- \( iPMT = \left((1 + \frac{r}{C})^{CP} - 1\right) \)

Investment Analysis

Lease vs. Purchase

- \( PMT_p \) = loan payment for purchase.
- \( PMT_L \) = lease payment.
- \( I_n \) = interest portion of \( PMT_p \) for period \( n \).
- \( D_n \) = depreciation for period \( n \).
- \( M_n \) = maintenance for period \( n \).
- \( T \) = marginal tax rate.
- \( \text{Cost of owning}(n) = PMT_p - T(I_n + D_n) + (1 - T)M_n \)

Break-Even Analysis and Operating Leverage

- \( GP \) = Gross Profit.
- \( P \) = Price per unit.
- \( V \) = Variable costs per unit.
- \( F \) = Fixed costs.
- \( U \) = number of Units.
- \( OL \) = Operating Leverage.
- \( GP = U(P - V) - F \)

\[
OL = \frac{U(P - V)}{U(P - V) - F}
\]
Profit and Loss Analysis

- Net income = (1 - tax)(net sales price - manufacturing expense - operating expense)
- Net sales price = list price(1 - discount rate)
- where operating expense represents a percentage of net sales price.

Securities

Discounted Notes

Price (given discount rate)

- \( B \) = number of days in year (annual basis).
- \( DR \) = discount rate (as a decimal).
- \( DSM \) = number of days from settlement date to maturity date.
- \( P \) = dollar price per $100 per value.
- \( RV \) = redemption value per $100 par value.

\[
P = [RV] - \left[ DR \times RV \times \frac{DSM}{B} \right]
\]

Yield (given price)

- \( B \) = number of days in year (annual basis).
- \( DSM \) = number of days from settlement date to maturity date.
- \( P \) = dollar price per $100 par value.
- \( RV \) = redemption value per $100 par value.
- \( Y \) = annual yield of investment with security held to maturity (as a decimal).

\[
Y = \left[ \frac{RV - P}{P} \right] \times \left[ \frac{B}{DSM} \right]
\]

Forecasting
Simple Moving Average
- $\Bar{X} = \text{moving average.}$
- $m = \text{number of elements in moving average.}$

\[
\Bar{X}_t = \frac{X_t + X_{t-1} + \ldots + X_{t-m+1}}{m}
\]

- etc.

Seasonal Variation Factors Based on a Centered Moving Average
- $\Bar{X}_c = \text{centered moving average}$
- $m = \text{number of elements in the centered moving average.}$

\[
\Bar{X}_c = \frac{X_t + X_{t-1} + \ldots + X_{t-m+1}}{m}
\]

- $SV = \text{Seasonal variation factor.}$
- $x_i = \text{value of the ith data point.}$
- $i = \text{centered moving average of the ith data point.}$

\[
SV = \frac{x_i}{\Bar{X}_c}
\]

Gompertz Curve Trend Analysis
- $y = ca^{bx}$
- where $x, y, a, b,$ and $c$ are positive.

\[
D_t = S_t + \frac{(1-a)}{\alpha} T_t
\]

\[
b = \left( \frac{S_3 - S_2}{S_2 - S_1} \right)^n
\]
\[ c = \exp \left( \frac{1}{n} \frac{S_1 S_3 - S_2^2}{S_1 + S_3 - 2S_2} \right) \]

\[ a = \exp \left( \frac{(b-1)(S_2 - S_3)}{b(b^n - 1)} \right) \]

Where \( S_1, S_2, \) and \( S_3 \) are:

\[ S_1 = \sum_{i=1}^{n} \ln y_i = n \ln c + b(\ln a) \frac{b^n - 1}{b - 1} \]

\[ S_2 = \sum_{i=n+1}^{2n} \ln y_i = n \ln c + b^{n+1} (\ln a) \frac{b^n - 1}{b - 1} \]

\[ \hat{D}_{t+1} = S_t + \left( \frac{1}{a} \right) T_t \]

\[ a, b \text{ and } c \text{ are determined by solving the three equations above simultaneously.} \]

Forecasting With Exponential Smoothing

- \( a = \) smoothing constant \((0 < a < 1)\)
- \( X_t = \) actual current period usage
Smoothed average $S_t = \alpha X_t + (1 - \alpha)S_{t-1}$

Change, $C_t = S_t - S_{t-1}$

Trend, $T_t = \alpha C_t + (1 - \alpha)T_{t-1}$

Current period expected usage,

Forecast of next period expected usage,

Error, $e_t = t - X_t$

Cumulative error $= \sum e_t^2$

Initial conditions: $S_{t-1} = X_{t-1}$
$T_{t-1} = 0$

**Pricing Calculations**

**Markup and Margin Calculations**

- $Ma = \text{margin}($\%$)$.
- $Mu = \text{markup}($\%$)$.
- $S = \text{selling price}$.
- $C = \text{cost}$.
\[Ma = 100 \frac{S - C}{S}\]

\[Mu = 100 \frac{S - C}{C}\]

\[S = \frac{C}{1 + \frac{Ma}{100}}\]

\[S = C \left(1 + \frac{Mu}{100}\right)\]

\[C = S \left(1 - \frac{Ma}{100}\right)\]

\[C = \frac{S}{1 + \frac{Mu}{100}}\]

\[Ma = \frac{Mu}{1 + \frac{Mu}{100}}\]

\[Mu = \frac{Ma}{1 + \frac{Ma}{100}}\]
Calculations of List and Net Prices with Discounts

- \( L \) = List price.
- \( N \) = Net price.
- \( D \) = Discount(\%).

\[
D' = 1 - \frac{D}{100}
\]

\[
L = \frac{N}{D_1 \times D_2 \times \ldots \times D_x}
\]

\[
D_x = \left(1 - \frac{N}{L(D_1 \times D_2 \times \ldots \times D_{x-1})}\right)
\]

Statistics

Exponential Curve Fit

- \( y = Ae^{Bx} \)

\[
B = \frac{\sum x_i \ln y_i - \frac{1}{n}(\sum x_i)(\sum \ln y_i)}{\sum x_i^2 - \frac{1}{n}(\sum x_i)^2}
\]

\[
A = \exp\left[\frac{\sum \ln y_i}{n} - B \frac{\sum x_i}{n}\right]
\]

\[
\hat{y} = -Ae^{Bx}
\]

Logarithmic Curve Fit

- \( y = A + B(\ln x) \)
\[ B = \frac{\frac{1}{n} \sum_{i=1}^{n} \ln y_i - \frac{1}{n} \sum_{i=1}^{n} \ln x_i \sum_{i=1}^{n} \ln y_i}{\sum (\ln x_i)^2 - \frac{1}{n} (\sum \ln x_i)^2} \]

\[ A = \frac{1}{n} \left( \sum_{i=1}^{n} y_i - B \sum_{i=1}^{n} x_i \right) \]

\[ \hat{y} = A + B (\ln x) \]

**Power Curve Fit**

- \( y = Ax^B \) (A > 0)
- \( \ln y = \ln A + B \ln x \)

\[ B = \frac{\left( \sum \ln x_i \right) \left( \sum \ln y_i \right)}{n} \]

\[ \frac{\sum (\ln x_i)^2 \left( \sum \ln y_i \right)}{n} - \frac{(\sum \ln x_i)^2}{n} \]

\[ A = \exp \left[ \frac{\sum \ln y_i}{n} - B \frac{\sum \ln x_i}{n} \right] \]

\[ \hat{y} = A x^B \]

**Standard Error of the Mean**

- \( S_{\hat{x}} = \frac{S_X}{\sqrt{n}} \)
- \( S_y = \frac{S_Y}{\sqrt{n}} \)

Mean, Standard Deviation, Standard Error for Grouped Data
Personal Finance

Tax-Free Retirement Account (IRA) or Keogh Plan

- \( n \) = the number of years to retirement.
- \( i \) = the compounded annual interest.
- \( PMT \) = the earnings used for investment (and taxes).
- \( FV \) = future value.
- \( tax \) = the percent tax expressed as a decimal.

For ordinary taxable investment:

\[
FV = \frac{PMT}{i(1-tax)} \left[ \frac{1 + i(1-tax)}{1 + i(1-tax)} \right] \left[ \frac{1 + i(1-tax)^n}{1 - 1} \right]
\]

For tax-free investment:

\[
FV = \frac{PMT}{i} \left[ \frac{1 + i}{1 + i} \right] \left[ \frac{1 + i^n}{1 - 1} \right]
\]

Stock Portfolio Evaluation and Analysis

- \( n \) = the number of issues held.
- \( P_j \) = the current market price / share of a stock.
- \( S_j \) = the number of shares of a stock held.
- \( \beta_i \) = the beta coefficient of an individual stock.
- \( T \) = the total present value of a portfolio.
Portfolio beta coefficient:

\[
\beta = \sum_{i}^{n} p_{i} \beta_{i}
\]

Canadian Mortgages

- \( r \) = annual interest rate expressed as a decimal.

- monthly factor = \( \left( 1 + \frac{r}{2} \right)^{\frac{1}{6}} - 1 \) \times 100

Miscellaneous

Learning Curve for Manufacturing Cost

- \( C_n \) = Cost of the \( n \)th unit.
- \( C_1 \) = Cost of the first unit.
- \( n \) = number of units.
- \( r \) = learning factor.
- \( k = \ln r / \ln 2 \)
- \( C_n = C_1 r^k \)
- \( \overline{C}_{ij} \) = the average cost of the \( i \)th through \( j \)th unit.

\[
\overline{C}_{ij} = \frac{C_1 \left[ \begin{array}{c} k \ast t - \frac{k + 1}{k + 1} \\ j - 1 \end{array} \right]}{1 - 1}
\]

- This formula is only approximate and may give appreciable error at small \( i \).
Queuing and Waiting Theory

- \( n \) = number of servers.
- \( \lambda \) = arrival rate of customers (Poisson input).
- \( \mu \) = service rate for each server (exponential service).
- \( \rho \) = Intensity factor = \( \frac{\lambda}{\mu} \) (\( \rho \), \( n \) for valid results).
- \( P_0 \) = Probability that all servers are idle.
- \( P_b \) = Probability that all servers are busy.
- \( L_q \) = Average number of customers in queue.
- \( L \) = Average number of customers in the system (waiting and being served).
- \( T_q \) = Average waiting time in queue.
- \( T \) = Average total time through the system.
- \( P(t) \) = Probability of waiting longer than time \( t \).

\[
\begin{align*}
P_0 &= \left[ \sum_{k=0}^{n-1} \frac{\rho^k}{k!} \frac{\rho^n}{n! \left(1 - \frac{\rho}{n}\right)} \right]^{-1} \\
\end{align*}
\]

\[
\begin{align*}
P_b &= -\frac{\rho^n P_0}{n! (1 - \frac{\rho}{n})} \\
\end{align*}
\]

\[
\begin{align*}
L_q &= \frac{\rho P_b}{n - \rho} \\
L &= L_q + \rho \\
T &= L / \lambda \\
T_q &= \frac{L_q}{\lambda} \\
P(t) &= P_b e^{(n - \lambda) t}
\end{align*}
\]

Graduated Payment Mortgage
where:

- \( A \) = number of payments per year
- \( B \) = number of years that payments increase
- \( C \) = percentage increase in periodic payments (as a decimal)
- \( PMT_1 \) = amount of the first payment